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ABSTRACT

A miniconference (consisting of four 60-minute sessions) was organized to contribute to the exploration of frames, forms, and contents of systematic cooperation and interaction between the researcher and teacher in the classroom. These proceedings of the conference include descriptions of the organization, purpose, and background of the conference, abstracts of presentations of six case studies, and a statement of some major questions which were presented for discussion at the concluding panel debate. The document contains five invited papers: (1) "Clinical Interviews Emmbedded in the 'Philosophy of Teaching Units'-A Means of Developing Teachers' Attitudes and Skills" (Erich C. Wittmann); (2) "Developmental Research on Paper and in Action" (Piet Verstappen); (3) "Working with Teachers to Implement a Problem Solving Approach with Pupils Aged 9-13 Years" (Leone Burton); (4) "Research and Teacher Training: Co-operative Development of Teaching Materials for Stochastics in Grades 5/6" (Gerd von Harten); and (5) "L'Enseignement de l'Enumeration: Etude de deux problemes pratiques et fondamentaux dans le cadre de la theorie des situations et du contrat didactique" (Guy Brousseau). The final part of the report contains the concluding plenary debate on the systematic cooperation between theory and practice in mathematics education. (JN)

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Bent Christiansen, Erich C. Wittmann, Mary G. Kantowski,
Piet Verstappen, Leone Burton, Gerd von Harten, Guy Brousseau

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Bent Christiansen, Erich C. Wittmann, Mary G. Kantowski,
Piet Verstappen, Leone Burton, Gerd von Harten, Guy Brousseau

MINI-CONFERENCE AT ICME 5

SYSTEMATIC CO-OPERATION BETWEEN THEORY AND PRACTICE IN MATHEMATICS EDUCATION

TOPIC AREA RESEARCH AND TEACHING

Adelaide, Australia, August 25-29, 1984

Royal Danish School of Educational Studies
Department of Mathematics
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INTRODUCTION

The relationships between didactical theory and teaching practice constitute a problem field of major interest within the didactics of mathematics (i.e. within mathematics education as a discipline). As it has become increasingly recognized that these relationships are of an indirect and complementary nature, there has been a rapidly growing international interest in analysis and investigation of the factual role of theory about mathematics education for the teaching/learning process and of ways in which it is attempted to mediate theoretical ideas and models to teachers.

At ICME 5, this problem field was treated within the Topic Area "Research and Teaching" at a miniconference on the theme "Systematic Co-operation between Theory and Practice in Mathematics Education". The four sessions of the conference were based upon six reports about research projects in which the potential of interaction between theory and practice (e.g. represented through interaction between researchers and teachers) had been investigated. The reports and the debates provided examples of new and constructive forms of systematic interaction and co-operation between the researcher and the practitioner and thus served to promote the overall purpose of the conference which was to contribute to the exploration of frames, forms, and contents of such *systematic* co-operative efforts.

It was my privilege to organize this conference in co-operation with six colleagues, who had accepted to present the above mentioned "case studies" (cf. p. 89). The activities within "our" Topic Area were closely related to those belonging to the Congress Theme Group on "Theory, Research, and Practice" and, especially, to the activities within the Topic Area "Theory of Mathematics Education (TME)" and at the Post-Congress Conference on TME *). The two latter programme events were organized by Professor H.G. Steiner, IDM Bielefeld, to whom I address my best thanks for his friendly assistance which made co-ordination of the TME-series and the Theory/Practice-series possible. Thus, the theoretical presentations in the former series by Steinbring and Brousseau served as background for the case studies presented in the latter by respectively von Harten and Brousseau. I similarly express my great appreciation to Mrs. Marjorie Carss (University of Queensland), Chairperson of the ICME 5 National Program Committee, for her kind and effective solution of the difficult problems created by the need to avoid conflicts between the time-tables of the three related programme events just mentioned.

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*) Steiner, H.G. et al.: Theory of Mathematics Education (TME) ICME 5 Topic Area and Miniconference: Adelaide, Australia, August 24-30, 30-31, 1984. Occasional Paper 54, IDM Bielefeld, 1984.

A brief report of the activities of the miniconference on "Systematic Co-operation between Theory and Practice" has been submitted for inclusion in the Congress Proceedings. It contains descriptions of the organization, purpose, and background of the conference, as well as abstracts of the six presentations of the case studies and a statement of some major questions which were presented by the undersigned for discussion at the concluding panel debate. This brief report is included here (pp. 7-17) because it provides quick access to an overview of the scope and contents of the conference. Next follow five of the invited papers as they have been submitted by the speakers. Finally, the present document brings an extensive report of the concluding debate, from which it will be seen that the seven colleagues - from six countries - who formed the constituent group behind the conference hoped to continue their co-operation and expected that their home institutions would support them in this process.

At the time of writing this introduction and submitting the report to printing, July 1985, it is my pleasure to state that preparations for future international co-operation concerning the investigation of the theory/practice interface are progressing in a very promising way. Colleagues, who are active in this field and would like to be kept informed about future developments, are invited to contact the undersigned.

Bent Christiansen

Topic Area: RESEARCH AND TEACHING

Chief Organizer and Reporter: Professor Bent Christiansen, Department of Mathematics, Royal Danish School of Educational Studies, Emdrupvej 115 B, DK 2400 Copenhagen NV, Denmark. Telephone: 01 - 696633 ext.2691. Co-organizers: Professor Guy Brousseau, IREM de Bordeaux, Dr. Gerd v. Harten, Institut für Didaktik der Mathematik (IDM) Bielefeld, and Professor Mary G. Kantowski, Subject Specialization Teacher Education, University of Florida.

1. Sub-Title: *Systematic Co-operation between Theory and Practice in Mathematics Education.*

2. Overall Purpose: To contribute to the exploration of frames, forms, and contents of systematic co-operation and interaction between the researcher and the teacher in the classroom.

3. Format: A miniconference of four sixty-minutes sessions, three used for presentation of cases of co-operation and the fourth for a concluding debate.

4. Background: Theory, research, and practice belong together.

Thus, didactical theory is - due to its very nature - concerned with the problem field of teaching and learning in the system of school. And the objects of didactical research (e.g. investigating, comparing, developing, or applying theoretical constructions) belong to, or are constituted in close relation to, that same field.

Accordingly, both *the initiation of research projects* in mathematics education and *the research processes* are conditioned by, and in varying degrees determined by, the flow of the teaching/learning process in the classroom and in school as a societal institution. And, finally, the importance of the relationships to school practice becomes evident in *the evaluation of the research results* and when questions are raised as to their significance for the teacher and their implementation into his practice.

However, the relationships between theory and practice are of an indirect and complementary nature, which makes mediation between these domains impossible by straightforward, pragmatic means. Hence, there is an increasing interest in the identification of links between theory and practice which can be utilized and exploited through new and constructive forms of interaction and co-operation between the practitioner and the researcher. It is the overall purpose of the miniconference to contribute to the exploration of frames, forms, and contents of such *systematic* co-operative efforts.

5. Relations to other Congress Activities: The program of this congress component has been developed in co-ordination with the preparations for the Topic Area "Theory of Mathematics Education (TME)" and is closely related to Theme Group 4 on "Theory, Research, and Practice". It is also linked to a *Post-Congress Miniconference* on TME held in Adelaide on 30-31 August 1984 heading at the establishment of working structures and plans for future work related to TME.

The papers read at the sessions on "Systematic Co-operation between Theory and Practice" and the proceedings of the debate at the final session will be published by the Department of Mathematics at the Royal Danish School of Educational Studies and will be available upon request from the chief organizer.

6. The programme and the proceedings

Session 1:

1a *Erich Wittmann* (Dortmund, FRGermany):
Clinical interviews embedded in the "Philosophy of Teaching Units". A means of developing teachers' attitudes and skills.

1b *Mary G. Kantowski* (Florida, U.S.A.):
Research and instruction in mathematics through problem solving in grades 7-12.

Session 2:

2a *Piet Verstappen* (Enschede, Holland):
Developmental research on paper and in action. Co-operation between teachers and researchers in the SLO-project "Mathematics 12-16".

2b *Leone Burton* (London, U.K.):
Working with teachers to implement a problem solving approach with pupils aged 9-13 years.

Session 3:

3a *Gerd v. Harten* (Bielefeld, FRGermany):
Research and teacher training: co-operative development of teaching materials for stochastics in grades 5-6.

3b *Guy Brousseau* (Bordeaux, France):
L'enseignement de l'énumeration: Etude, dans le cadre de la theorie des situation et du contrat didactique, de deux problemes pratiques et fondamentaux.

Session 4:

At this session major issues identified during the previous sessions were presented by *Bent Christiansen* and under his chairmanship discussed in a panel consisting of the six speakers and *Nicholas Balacheff* (Grenoble, France).

Below follows a brief report of the six presentations and of the panel-session. Each presentation was succeeded by questions and answers which unfortunately cannot be included in the report.

Report of Sessions 1-3:

1a ERICH WITTMANN took as his opening point the teacher's need for models of the more or less open situations by which he is confronted in his daily practice. The teacher develops such a teaching model of a given situation by means of his/her *theoretical repertoire* supported by experience and existing teaching aids. Such a teaching model tells the teacher what to look for, what to do, what to expect. And it may also provide reasons for the teacher's observations, decisions, and prognoses. The personal experience with such models (which are being revised during the teaching process) will over time influence the teacher's theoretical repertoire, which may be seen as *a collection of subjective theories*. Using Popper's conception of "the three worlds", the teacher's *practice* belongs to world 1, his *theoretical repertoire* to world 2, whereas *theory* in the sense of didactical theories about teaching is part of world 3. On this background

Wittmann stated the central issue of teacher education as the question: what is the best way to build up an effective theoretical repertoire for teaching? He commented briefly upon two well-known answers: (1) the practitioner's view - *through practice*; (2) the theorist's view - *through theory*; and then treated in more details (3) John Dewey's answer (of 1904) - *through "vital theoretical instruction"*. Wittmann summarized and re-phrased Dewey's position as follows: "The main task of a teacher is to stimulate and to develop the mental activity and interaction of his or her pupils. The best way for a teacher student to acquire the necessary competence is to develop methods of mathematical thinking, to reflect upon them, to observe and analyse his or her own learning, in interaction with other students, and to study mathematical thinking in children and groups of children". And Wittmann introduced in this perspective his *own* position: "This kind of *doing mathematics* and *doing psychology* shares the essential aspects of learning and teaching mathematics in the classroom. So it represents some sort of practice, which I would like to call *"intermediate practice"*. To me intermediate practice is the ***keystone*** of relating theory and practice in education to each other."

Wittmann next gave a brief description of the concept *teaching unit* which he had recently treated in more detail (in ESM, 1984). He proposed that didactical research and development, and also teacher education, should be related to groups of teaching units which allow for mathematical and psychological activities in the sense of intermediate practice and which are representative of the curriculum.

Finally, he described ways in which *clinical interviews* had been used in teacher education programmes in Dortmund during the last decade. The themes are now taken from teaching units which are subjected to field-based exploration, and the interviews are performed in co-operation with kindergartens and primary schools. Themes, content, and form of interviews performed by student teachers were exemplified, and Wittmann concluded that clinical interviews - under the conditions mentioned - in his opinion: (1) constitute the easiest way of doing psychology; (2) foster attitudes and skills of good teaching; and (3) relate psychology to other domains of mathematics teacher education.

1b MARY KANTOWSKI described a study at the College of Education, University of Florida, investigating the role of instruction for the development of students' ability to solve nonroutine tasks (*problems*). The research took place during a teaching experiment in which teachers in-service co-operated with a team of researchers, and the rationale behind the study had two aspects: to promote the teachers' understanding of the educational problem field to be investigated through the co-operative efforts; and to promote that the findings of the study would become internalized by the teachers as useful tools.

The theoretical basis comprised: (1) *theory of problem solving* (e.g. information processing theory and Polya's four phases); and (2) *theory of instruction for problem solving* (concerning the relations between the teacher's role and functions and the student's personal activity). To become good problem solvers, students must first learn what constitutes the process of problem solving and how content and process interact in producing solutions to nonroutine mathematical tasks. But furthermore, instruction must be planned so that students are provided with the tools or "heuristics" for effective problem solving without having creativity impaired. Mary Kantowski described how teachers changed their role during the project from "model" via "supporter" to "facilitator", and how students took on a more active role in finding solutions as problem solving ability increased. Clinical interviews of problem solving students had been performed by the team of researchers to provide data for support of the teachers.

Form, extent, and content of the co-operation had depended strongly on the teacher's background. All teachers participated in observation and collection of data. Teachers with limited experience had made no input into the selection and investigation of problems. Teachers with moderate experience had been involved in coding and interpretation of data and had provided some input to the planning for instruction. And experienced teachers had been active in the selection of problems and in the planning of their own teaching and of the gathering, coding, and interpretation of data.

Finally, some general results were mentioned: The great variety of students' solutions had been remarkable; it seemed clearly indicated that there is a need at school level for instruction in heuristics; there had been a marked, general need for an "incubation period"; and the experimental course had produced greatest gain in "average" students. As results concerning the working with teachers were mentioned that there is a need for progressive types of teacher's experiences, and a general need for continuing experiences in research of this co-operative kind.

2a PIET VERSTAPPEN pointed in his opening remarks to the specific conditions for didactical research in the Netherlands, under which curriculum development has been centred in one institute, whereas research and teacher education have been settled elsewhere. He would report on the project "Mathematics 12-16" based at the National Institute for Curriculum Development (SLO: Stichting voor de Leerplanontwikkeling), in which six developers of the SLO, three researchers at the University of Utrecht, and twelve teachers and teacher trainers co-operate. The project has been in operation since 1981. Its major goal is the development of an overall curriculum design for mathematics teaching at age level 12-16, and Piet Verstappen characterized the perspectives and starting points of the project by means of brief statements: - mathematics for all; - relating mathematics to everyday life; - presenting mathematics from contexts corresponding to the pupil's intuitive knowledge; - relating mathematics to the individual's own approach; - respecting the right of learning with insight for all. The theoretical objectives of the group of researchers were summarized as follows: to contribute to the development and improvement of the learning materials; and to clarify the concept "mathematics for all".

Piet Verstappen commented upon different types of research and characterized (referring to the systematics of Terwel) the research belonging to the project as *education developmental research*, which in short can be described as an informal, incremental, developing approach with strong emphasis on improvement of educational practice and development of learning materials. The researcher has many roles (developing, training, supporting, investigating). He may interfere and influence spontaneously and plays an important part in the dissemination of results and convictions of the various groups. The developer's task is markedly creative (designing new learning materials and testing these for usefulness). Accordingly, he works in school, where he gives lessons and participates in the teaching process. His role is on the one hand to analyse, to improve, and to report his findings concerning the arrangement of the mathematics lessons, and, on the other hand, to be an inspiration for teachers, teacher trainers, and authors of textbooks.

The problem field in consideration at the present mini-conference had been narrowed to the systematic co-operation between theory and practice. But there are in addition to external relationships between theory/research and practice, *internal* relationships in research; particularly the relation *developer-researcher*. Piet Verstappen described the experience obtained in the project concerning this relation and the relations *developer-teacher* and *researcher-teacher*. His description focussed for each relation on differences: (1) in viewpoints and conceptions; (2) in what is seen as problems; and (3) in communication. Gaps had been experienced, e.g. emanating from different opinions about theory, but Piet Verstappen proposed that teachers, developers, and researchers understand each other in the light of practice and that communication is promoted in relation to the child in the classroom. He emphasized the need for further exploration of the relations belonging to the triangle of teacher, developer, and researcher.

2b LEONE BURTON described a project funded by the Social Science Research Council which had investigated the feasibility of a problem-based approach to mathematics teaching of 9-13 year-old pupils. Behind the project were two related concerns. A mathematical concern to address the relationship between content and process and to attempt to improve the definition of mathematics as a problem-based discipline. And a corresponding pedagogical concern to find means whereby teachers and students could experience problem solving as part of their mathematics curriculum. The overall aim was: for teachers, to identify a structure which would support changes in pedagogical role whereby insight into pupils' mathematical experience could be gained (to encourage the teacher as a researcher in the classroom); and for pupils, to build confidence, reinforce problem solving skills and procedures, and change perception of mathematics. And here, the intention was rather to investigate if problem solving processes already exist in young learners and if encouraging their use changes the learning context, than to test the acquisition of processes.

The project team developed a bank of thirty problems for the pupils and supporting materials for the teachers. Teachers in five areas of England were invited to try out this pack of materials in collaboration with the research team, and 27 classes became involved. Each teacher was requested to devote at least one hour per week for one term to the problem solving and to perceive his role in this work rather as a resource than as an informant. Leone Burton described the co-operation between the teachers and the team. It was in each area opened by a one day meeting (at the end of the preceding term) which included group work on problems from the bank and subsequent discussion of the project materials, the background philosophy, classroom organization, etc. Pupils would work in small groups and have free choice of problems from the bank, and teachers were encouraged to withdraw problems that they considered unsuitable for their class. At all times, *responsibility for the class rested with the teacher* (although team members regularly visited the classes), whereas *responsibility for the work rested with the pupils* (if practicable).

Leone Burton described how results were recorded and evaluated. For the pupils, a substantial change was seen by the end of the term from early dependency on the teacher towards a flexible and independent approach, and the enjoyment of the work had been evident. For the teachers, a change took place from accepting problem solving activity as a teaching experiment to seeing it as a necessary component of the teaching/learning process. The study confirmed that such programmes can be implemented by non-specialist teachers and that their feasibility is not affected by type of school, location, or ability range. However, considerable support had been provided in personnel and materials, and teachers should not be expected to undertake such radical shift in style of work on their own *even* when materials are available. Future research might investigate: (1) a developmental approach to the recognition and use of problem solving processes; and (2) ways to simplify the theoretical background of the projects.

3a GERD VON HARTEN reported about the project LEDIS (Lehrplan und Differenziehrung in der Sekundarstufe I), which is carried out at IDM Bielefeld. The overall aims of the project are: (a) to obtain general results about appropriate forms of didactical materials for teachers; and (b) to develop teaching materials and assistance for teachers of stochastics at grade levels 5/6 of the comprehensive school of Nordrhein-Westfalen. The project seeks to obtain these aims by means of a co-operation with teachers which combines *further education in the subject* with *classroom experience*. Earlier research projects in F.R.G. had come to more or less negative comclusions about the value of linking curriculum development and in-service education.

But Gerd van Harten proposed that reforming the curriculum is of less concern for teachers than the daily problems of school and classroom, and that to overcome the separation of the subject matter from the social aspects of teaching and learning is of central importance for the development of a more independent school mathematics. However, all these projects have shown that interaction between researchers and teachers is highly difficult. In this perspective, LEDIS was to be seen not as a solution, but as a possible new approach attempting to integrate the mathematical subject matter, the organization of the teaching, the theoretical and educational background, and the form of the teaching materials.

Major ideas and theses behind the project were presented, e.g.: - the type of co-operation between researchers and teachers must be decided according to the content in question; - the didactical analysis of the mathematical content must prepare for relating the educational/mathematical subject matter aspects with the interactive/pedagogical/organizational social aspects of teaching; - the core of materials for mathematics teachers should consist of tasks, because teachers at this level plan lessons by means of tasks and not concepts; - in-service education must link subject matter with classroom experience; - these projects cannot be evaluated in the classical way; it is much more important to establish groups of teachers working continuously with the problem field and that the project materials allow for continual adjustments according to the individual teacher's needs and experience.

A central problem was how the theoretical and innovative intentions could be related to the teachers' experiences. A first version of the materials was developed by Gerd von Harten and H. Steinbring comprising: (1) a didactical analysis describing the didactical intentions of the lesson sequence and giving an overview of concepts and definitions for stochastics on the secondary I level; and (2) a system of tasks for stochastics in grade 5/6. This system constitutes an offer of opportunities for the teacher. It is attempted that each task in some sense is a subsystem reflecting the whole system and that the tasks contain ideas and requirements which permit manifold alterations. Hence, they may be simplified, or made more difficult, or omitted altogether, and thus there are different possible paths through the system. This first version was used by all teachers of mathematics at level 5/6 at five schools, and the work of 26 of these teachers was carefully observed and studied in the project. The experience gained confirmed the usefulness of materials of this form. It is now used by teachers at other comprehensive schools. A *final* version is not intended, since the material is to remain a teacher's "working book" to be continually altered and improved by teaching experience.

3b GUY BROUSSEAU mentioned that he had given a presentation in the morning session of the series "Theory of Mathematics Education" concerning the role of *the didactical contract* in the analysis and construction of *didactical situations*. His present contribution would exemplify how these two theoretical concepts were of support for practice. In brief, a didactical situation is a set of relations established explicitly and/or implicitly between: (1) a pupil or a group of pupils; (2) some part of the environment (including instruments or materials); and (3) the teacher, with the educational purpose to provide for the pupil's acquisition of some definite knowledge. And a didactical contract is a set of rules which organize the relation within the classroom between the content taught, the pupils, and the teacher (and which are normally not stated explicitly). He had chosen *counting* (in mathematical terms: the establishing of an injection onto an initial segment of the ordered set of natural numbers) as the example. This mathematical process appears at all levels of mathematics teaching - at school through university (e.g. in combinatorics) - and seems to be involved in numerous difficulties for the individual. However, counting (l'énumeration) is at no moment treated as an *object for teaching* in the sense that it becomes the core of intensive negotiations in the context of a complex didactical contract.

Thus, two fundamental problems of school practice were to be considered: (1) the identification of didactical situations which can serve to generate appropriate activity and questions to be used in the process of interaction and negotiation in the classroom as vehicles for the intended learning; and (2) the communication to the teacher of such situations together with appropriate knowledge and know-how about their potentials as well as of the involved didactical contracts.

The solution proposed in the project at IREM Bordeaux consisted in presenting an identified *fundamental didactical situation* for the teacher at the computer screen thereby providing him with access to each moment of the ongoing process of the child's activity and interaction with the objects displayed in a meaningful context at the screen. Details were given about the way in which the computer programme served to provide a scenery motivating pre-school children for activity (in this case involving actions with rabbits and ducks) with high potentials for learning related to counting. The child using an optical pencil was in continual interaction with the movable animals, and the teacher was thus enabled to observe and realize the role and function of the child in the developing process and to reflect upon and participate in this dynamic procedure. Especially, the programme provided opportunity for the teacher to change the nature of the didactical contract, for instance in order to counteract the classical danger of destroying or deleting - by his very teaching actions - vital meaning from the child's conception of the situation. Guy Brousseau identified the didactical variables of this fundamental situation and pointed finally to the capacity of this type of didactical analysis (which here had been used in answering the two questions raised in the example) for improving the contribution of didactical theory to practice.

Report of Session 4:

Bent Christiansen chaired a panel of the six speakers and *Nicholas Balacheff*, who had submitted questions in writing to two of the contributions. Christiansen had selected major issues for discussion based upon the preceding activities. He briefly mentioned the rationale behind the series and introduced in the course of the session the following three questions:

(I) What is the role of didactical theory?
    (1) for the researcher in his interaction with the teacher?
    (2) for the teacher in his interaction with his students?

(II) What are the potentials of such "interactive" projects:
    (1) in teacher education (pre-, in-service, and further)?
    (2) in curriculum development?

(III) Means to promote international co-operation within the field given by the theme of the mini-conference taken in the broad sense?

An extensive record of the proceedings of the session is included in this report at pp. 77-88.

CLINICAL INTERVIEWS EMBEDDED IN THE "PHILOSOPHY OF TEACHING UNITS"

- A means of developing teachers' attitudes and skills -

ERICH CHRISTIAN WITTMANN

INSTITUT FÜR DIDAKTIK DER MATHEMATIK
UNIVERSITÄT DORTMUND - F.R.G.

> Those who like practice without theory are like seamen sailing without steering-wheel and compass and never being sure where the voyage goes. Practice must always rest on good theory.
>
> Leonardo da Vinci

In mathematics education there is at present a clearly increased interest in methodological issues. This fact cannot be explained only by internal motives but it is also a defensive reaction to outside pressures from well-established disciplines as well as from teachers' associations which question the academic status and the utility of mathematics education. The relationship between theory and practice lies at the very heart of the problem. Thus it is to be expected that first and foremost the elaboration of effective ways to relate theory and practice to each other will help to define the specificity of mathematics education, to prove its value, and thus to stabilize it, internally and externally.

The teory-practice-relationship is more or less a problem in all applied fields of knowledge. So mathematics education can learn from experiences in more advanced disciplines which indicate the following general facts:

(1) The delineation of theory from practice is a natural and necessary step of development and, in principle, opens the way to more effectiveness.

(2) The relationship between theory and practice cannot be fixed once and for all, but it has to be continuously re-thought during the progressive development of the subject.

(3) Tensions between theory and practice are not bad in themselves, but they may be used for mutual criticism and thus they may serve as the source of progress.

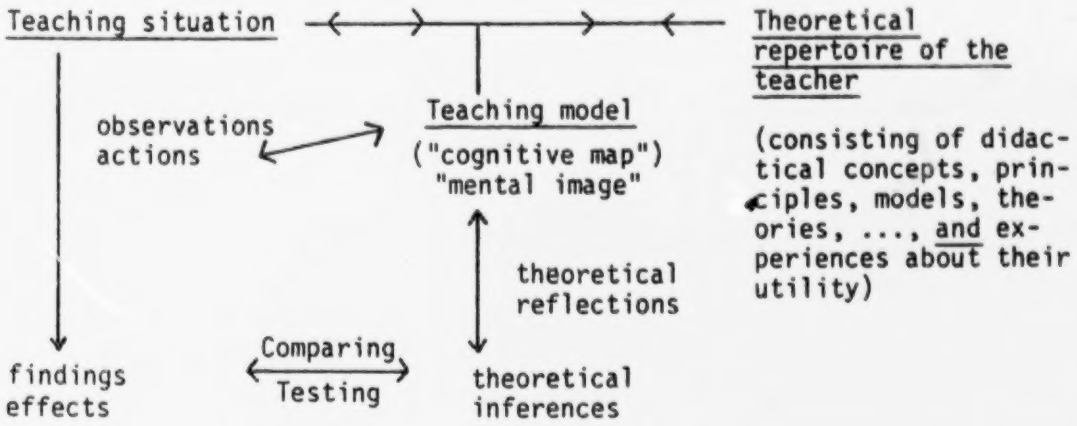
(4) There is always the tendency that theories will state their own ends and develop for themselves. This is no danger as long as they are related to a *significant core* which itself is related to practice in a vital way. If, however, theories develop in complete isolation from practice they are bound to become useless and to degenerate.

At the present stage of mathematics education the creation of such a significant core must be the primordial aim. I agree completely with the view expressed by Alan Bell in a forthcoming book review: "One might ask the general question whether, in the present state of knowledge about mathematical education, we should progress faster by collecting 'hard' data on small questions, or 'soft' data on major questions. It seems to me that only results related to fairly important practitioner questions are likely to become part of an intelligible scheme of knowledge. The developing theory of mathematical learning and teaching must be a refinement, an extension and a deepening of practitioner knowledge, not a separate growth. Specific results unrelated to major themes do not become part of communal knowledge. On the other hand 'soft' results on major themes of they seem interesting and provocative to practitioners, get tested in the myriad of tiny experiments which teachers perform every day when they "try something and see if it works"."

The present paper is based on an approach trying to bridge the gap between theory and practice by means of teaching units (Wittmann 1984). Although this approach relates to didactics as a whole, the perspective taken in this paper is that of pre-service teacher training. The first part will describe a particular view of the relation of theory to practice due to John Dewey (1904). The second part will sketch the "philosophy of teaching units". The third and essential part will show how clinical interviews can be used to develop both theoretical and practical skills of teaching.

1. Co-operation between theory and practice through "intermediate practice"

From the point of view of the practitioner the theory-practice interaction takes place on a small scale according to the following diagram which is adapted from the well-known theory-practice loop in science and which reads as follows:


Teaching situation
Theoretical repertoire of the teacher
(consisting of didactical concepts, principles, models, theories, ..., and experiences about their utility)
observations
actions
Teaching model
("cognitive map")
"mental image"
theoretical reflections
findings
effects
Comparing
Testing
theoretical inferences

Within his/her daily practice the teacher is continuously confronted with (more or less open) teaching situations. In order to manage a given situation he or she uses his/her theoretical repertoire, his/her experience and various aids for developing a model which tells him/her what to look for, what to do, what to expect, and which may also give reasons for his/her observations, decisions and prognoses. The model is a model "in the making", i.e. it is being revised during the teaching process.

The experiences with teaching models will in the long run lead to strengthening, weakening, modifying and revising ideas of the theoretical repertoire. So the theoretical repertoire itself is undergoing a continuous process. By its very nature the theoretical repertoire may be called a *subjective theory* (or perhaps better, a collection of subjective theories) of the teacher. It must be distinguished from the theories of teaching developed within mathematics education. Using Karl Popper's conception of the three worlds (Popper 1972) we can say: The teachers' field of activity belongs to world 1, his subjective theory of teaching to world 2, whereas didactical theories of teaching are part of world 3.

The central issue of teacher training is the following question: What is the best way to build up an effective theoretical repertoire for *teaching?*

One answer given for centuries and shared by the vast majority of practitioners even today is this (cf. Egsgard 1978): Suppose the prospective teacher knows his/her subject matter. Then the necessary theoretical tools evolve best through practice itself under the guidance of experienced teachers.

A second view held by many theorists sees the best professional preparation of teachers in a study of mathematical, educational and didactical theories accompanied or followed up by practical work.

Among the teacher training programs which at present exist around the world there are many examples for each of the two views.

Any evaluation of the two positions can only be made with respect to basic normative assumptions on mathematics teaching. I will follow a "genetic" perspective which is deliberately chosen in many departments of mathematics education and which may be characterized in this way:

(1) Mathematics is not just a collection of concepts, procedures and structures but a living organism which develops and grows under the stimulation of big and small problems inside and outside of mathematics.

(2) Knowledge cannot be simply transmitted, but it must be developed ("constructed") through the learner's own activity.

(3) Social interaction is an essential component of learning and development.

Although the genetic view has spread more widely only in recent times, its origins go far back in history and it received strong attention at the beginning of this centrury. Therefore it is not so surprising that the relation between theory and practice from this point of view was already elaborated by John Dewey, namely in a fundamental paper written in 1904. This paper seems to be relatively unknown. Therefore while reviewing it below I will give lengthy quotations.

Dewey sees the essential task of a teacher in "directing the mental movement of students" and stimulating the "interaction of mind": "As *every teacher knows, children have an inner* and an outer attention. The inner attention is the giving of the mind without reserve or qualification to the subject in hand. It is the first-hand and personal play of mental powers. As such it is a fundamental condition of mental growth. To be able to keep track of this mental play, to recognize the signs of its presence or absence, to know how it is initiated and maintained, how to test it by results attained, and to test *apparent* results by it, is the supreme mark and criterion of a teacher. It means insight into soul-action, ability to discriminate the genuine from the sham, and capacity to further one and discourage the other" (p. 148).

Dewey rejects the assumption held by the proponents of the apprenticeship-type of teacher training that the attitudes and skills of a good teacher can be acquired best through practice. On the contrary he considers premature practice as detrimental, because it puts the attention of the student-teacher in the wrong place, and tends to fix it in the wrong direction, namely towards controlling the external attention of children, towards keeping them fixed upon his/her own questions, suggestions, instructions and remarks and upon their "lessons".

According to Dewey reasonable practical training of teacher students is only possible "... where the would-be teacher has become fairly saturated with his subject matter, and with his psychological and ethical philosophy of education. Only when such things have become incorporated in mental habit, have become part of the working tendencies of observation, insight, and reflection, will these principles work automatically, unconsciously, and hence promptly and effectively. And this means that practical work should be persued primarily with reference to its reaction upon the professional pupil in making him a thoughtful and alert student of education, rather than to help him get immediate proficiency" (p. 150).

Dewey's own approach, which he calls "laboratory points of view", consists in forming the prospective teacher through "vital theoretical instruction". At first sight one might believe this approach would coincide with the second view mentioned above. However, Dewey is far from understanding "vital theoretical instruction" as *transmitting* mathematical, educational or didactical theories. So his position is in fact a *tertium*. It is characterized by a very subtle analysis of what mathematics and the educational disciplines may contribute to a teacher's subjective theory of teaching. As regards mathematics the important thing for him is not the bulk of ready-made structures but mathematical thinking: "There is therefore, method in subject matter itself - method indeed of the highest order which the human mind has yet evolved, scientific method.

It cannot be too strongly emphasized that this scientific method is the method of mind itself ... (It) reflect(s) the attitudes and workings of mind in its endeavour to bring raw material of experience to a point where it at once satisfies and stimulates the needs of active thought. Such being the case, there is something wrong in the "academic" side of professional training, if by means of it the student does not constantly get object-lessons of the finest type in the kind of mental activity which characterizes mental growth and, hence, educative process.

It is necessary to recognize the importance for the teacher's equipment of his own habituation to superior types of method of mental operation. The more a teacher in the future is likely to have to do with elementary teaching, the more, rather than the less, necessary is such exercise. Otherwise, the current traditions of elementary work with their tendency to talk and write down to the supposed intellectual level of children, will be likely to continue. Only a teacher thoroughly trained in the higher levels of intellectual method and who thus has constantly in his own mind a sense of what adequate and genuine intellectual activity means, will be likely, indeed, not in mere word, to respect the mental integrity and force of children" (p. 160-161).

As regards the educational training of teachers Dewey takes again the perspective of the teacher concentrated on the mental growth of children and arrives at a surprising conclusion: "What the (teacher) student needs most at this stage of growth is ability to see what is going on in the minds of a group of persons also who are in intellectual contact with one another. He needs to learn to observe psychologically - a very different thing from simply observing how a teacher gets 'good results' in presenting any particular subject (p. 156) ... .

It is not too much to say that the most important thing for the teacher to consider, as regards his present relations to his pupils, is the attitudes and habits which his own modes of being, saying, and doing are fostering or discouraging in them.

Now ... I think it will follow as a matter of course that only by beginning with the values and laws contained in the student's own experience of his own mental growth, and by proceeding gradually to facts connected with other persons of whom he can know little; and by proceeding still more gradually to the attempt actually to influence the mental operations of others, can educational theory be made most effective. Only in this way can the most essential trait of the mental habit of the teacher be secured - the habit which looks upon the internal, not upon the external; which sees that the important function of the teacher is direction of the mental movement of the student, and that the mental movement must be known before it can be directed" (p. 158).

I would like to summarize and re-phrase Dewey's position as follows: The main task of a teacher is to stimulate and to develop the mental activity and interaction of his or her pupils. The best way for a teacher student to acquire the necessary competence is to develop methods of mathematical thinking, to reflect upon them, to observe and analyse his or her own learning, in interaction with other students, and to study mathematical thinking in children and groups of children. This kind of *doing mathematics* and *doing psychology* shares the essential aspects of learning and teaching mathematics in the classroom. So it represents some sort of practice, which I would like to call *"intermediate practice"* To me intermediate practice is the *keystone* of relating theory and practice in education to each other.

From the preceding it should go without saying that theoretical studies of mathematics, psychology and education in the sense of intermediate practice are not possible under separation of the disciplines but require an interdisciplinary approach. Additionally, new forms of organizing teacher training are necessary. I think here is the chance for mathematics education.

2. The "Philosophy of Teaching Units"

The importance of intermediate practice for classroom teaching would be increased if it were possible to relate doing mathematics and doing psychology to the school curriculum without, however, trivializing or distorting the requirements mentioned before. Wittmann (1984) suggested to center didactical research and development as well as teacher training around groups of teaching units which are sufficiently rich in order to allow for mathematical and psychological activities in the sense of intermediate practice and which are representative of the curriculum. Of course this idea is not completely new. E.g. it is developed to some extent in the well-known book "Some lessons in mathematics" written by members of ATM (Fletcher 1965). What seems to be new, however, is the *systematics* of the approach.

Steps in elaborating this "philosophy of teaching units" for the primary level are Müller/Wittmann 1984 and Wittmann 1982.

3. Clinical interviews within the "philosophy of teaching units"

I would like now to focus on one aspect of the philosophy of teaching units, namely a specific use of clinical interviews.

The clinical method and other protocal methods are enjoying increased popularity among researchers in mathematics education. (Easley 1977, Ginsburg 1983). Bergeron and Herscovics (1980) have suggested to use clinical interviews also in teacher training.

In Dortmund we started a course "Development of mathematical thinking" as part of our teacher education programs about 10 years ago. First this course was more or less an introduction into Piagetian psychology. Clinical experiments played an important but nevertheless subordinate role in the course. They were used to illustrate Piaget's theory. The first interviews done by the teacher students themselves were nothing but replications or smaller variations of experiments from the Genevan school.

During the years the make-up of the course has changed considerably. Firstly the pitfalls, inconsistencies and flaws of Piaget's stage theory became more and more apparent to us, something which happened all over the world (cf. Brown & Deforges 1979, Groen/Kieran 1983). Our reaction was to emphasize the clinical interview, Piaget's research method, and to push his theory to the background. At the same time we realized that the time limit for the course didn't allow for a lengthy introduction into psychology anyway. So, we decided to concentrate the psychological-didactical training of our students on "doing psychology" in analogy to the mathematical training which we had based on the concept of "doing mathematics" before. It seemed to us that clinical interviews are the easiest way of doing psychology. As a consequence the course was split up into two parts:

(1) an introduction into the clinical method and into basic ideas of Piaget's genetic epistemology,

(2) conducting clinical interviews with kindergarten or primary children.

The second part, the seminar, turned out to be extremely motivating for teacher students, and revealed the fact that clinical interviews are very valuable instruments for developing attitudes and skills of good teaching far beyond the psychological insights they may develop. Such virtues of good teachers are: introducing children to a situation, making them feel comfortable, letting them work and observing them without interrupting, showing interest, listening carefully, accepting the children's thinking, avoiding criticism or authoritarian evaluation of the children's ideas, stimulating their thinking by cautiously arousing cognitive conflicts or by pointing to facts and statements seeming to be overlooked, etc.

Accordingly our second change in the course consisted in stressing the pedagogical aspects of clinical interviews.

The third change at last referred to the contents of the interviews. As already mentioned we started by replicating Genevan studies. These, however, are far removed away from the mathematics curriculum, in particular with respect to thinking processes. So we replaced them more and more by themes which are taken from teaching units. I would like to give an impression by describing some themes through the problems which "define" them.

(1) Even and odd numbers

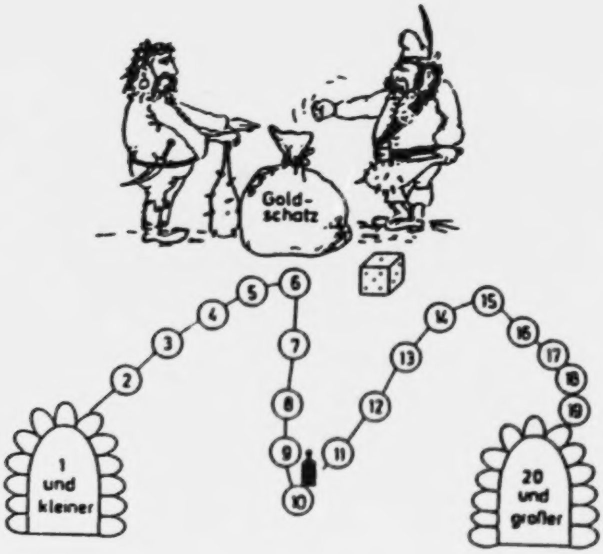
Can you tell me even and odd numbers?
Is 16 even or odd? Why?

Given a set of counters: How can you find out if the number of these counters is even or odd?

The child is told the parity of each of two given sets of counters, but the exact numbers are not told: Will the whole be even or odd when I put the two sets together? Why?

(2) The robbers and the treasure

Two robbers are fighting for a treasure. After some time there is no winner and they are exhausted. So they agree to decide the quarrel by playing a game: They paint fields 1 to 20 between their caves. The treasure is put on field 10. Now they throw the die alternately. According to the result the treasure is moved towards the corresponding cave. As soon as the treasure enters a cave the owner of the cave wins it.



Play the game with your partner!

Suppose the treasure is on field 11: Where might it be after each of the two robbers has thrown the die just once?

Where will the treasure be if the right robber throws a "5" and the left robber a "4"?

(3) Only nine figures

Out of the nine figures 1,2,...,9 form three 3-digit numbers such that their sum is as large as possible!

Why do you think your sum is maximal? Can you find other solutions?

How to make the sum as small as possible?

(4) A word problem

In a class of 29 children there are two more girls than boys.

How many girls and boys are there?

(5) Ice-cream

An ice-cream seller offers four kinds of ice-cream: chocolate, lemon, raspberry, pistachio. He sells cones with three scoops. How many different cones are possible?

(6) Games of strategy

Given a game of strategy: Play the game with your partner and find out how to play as clever as possible!

I do hope that the reader can imagine both what clinical interviews (and what teaching models) about the themes (1)-(6) would be like.

The themes for clinical interviews are in part stimulated by theoretical considerations. One would just like to know how children of different ages will react to this or that problem in order to use that knowledge for teaching. In many cases, however, the inspiration comes the other way round from observing classes. One sees some interesting point which is too fast swept away by the interactions in the classroom and one wishes to learn more about it, to grasp the children's thinking in more detail. E.g. theme (2) is due to a very fine lesson. I have seen given by a primary teacher to third-graders.

So there is a renewing queue of themes which is presented to the participants of every new seminar. Any group of two students chooses one theme according to their taste and prepares a clinical interview.

Depending on which age level seems interesting the group applies to a kindergarten and/or primary school, explains to the nurses and/or teachers what it is planning, and asks for collaboration. Supported by the nurses and/or teachers the intended interview is conducted with some 15 children. Afterwards the interviews are transcribed and analysed, a report is written and discussed in the seminar.

Some of the students extend their clinical study into a paper as part of the final examination. E.g. a student is presently working on theme (2) in co-operation with two schools. The teachers of grade 1 have identified children who have difficulties in adding and subtracting numbers. The student uses the game both as a diagnostic and a remedial instrument. An interesting theoretical question here is the transition from material-bound operations to mental calculations. At the same time the study will give information about the utility of the game as a context for practising computational skills. This kind of co-operation with schools looks quite promising and we are going to extend it.

Our experiences with the course in its new format are as follows. Firstly the course serves its purpose as a framework for intensive intermediate practice. Clinical interviews with individual children or very small groups of children in kindergarten or primary school represent a protected atmosphere where a teacher student may concentrate on "intellectual contact", "interaction of mind" and "mental movement", to use John Dewey's terms. The students are also stimulated to reflect on their own behaviour and its influence on children. With some students this results in quite dramatic changes of awareness. In retrospect from the practical phase of teacher training - which in Germany is spent in school and takes two years - the students evaluate the relevance of the course very highly. The course ranks first ahead of all other mathematical, didactical, and educational courses.

Our second experience is that the skill of students in conducting clinical interviews is a very good indicator of their skill in teaching a class. This is not surprising, because it is the attitudes and skills of good teaching that are required with clinical interviews as has already been mentioned above.

4. Concluding remarks

I can easily imagine that some colleagues, who are more in favour of major themes, basic ideas, and great lines will find the occupation with teaching units conceptually poor, too little controlled, not "research-oriented" and, in fact, the whole approach naive. I would like to say the following.

Firstly, I think teaching units are a natural way to achieve an integral view of mathematical, psychological, pedagogical, and practical aspects of mathematics teaching which I see as the specificity of didactics of mathematics.

Secondly, I do not plead for losing oneself in isolated teaching units, but for seeing them as a representation of objectives, contents and principles of mathematics teaching and as medium of teaching practice. E.g. among the six themes in the previous section (2) and (3) are examples of a new approach to practising skills (Winter 1984, Wittmann 1984), (1) and (6) are typical for studying and developing mathematical processes (cf. Bell 1979), (5) belongs to a class of units for studying and developing combinatorial thinking and (4) fits into the network of investigations on word problems. All themes could be used to study social interaction in the classroom. For bridging the gap between theory and practice I think it is necessary to have *realistic* empirical tests of theoretical ideas. Why not use teaching units for this purpose which may lend theoretical ideas 'meaning' and 'significance'? A third reason for centering research, development, and teacher training around teaching units is that teaching units offer excellent opportunities for intermediate practice and thus for shaping effective subjective theories of teaching (cf. section 1). Additionally, the ability to do mathematics and to do psychology is in my opinion an essential prerequisite for making use of didactical theory. The past decades have provided us with clear evidence that there is no one-to-one transmission from theory to practice. Almost everything depends on adaptive teachers equipped with heuristic strategies for supplementing, selecting, modifying, rearranging, specializing, transferring, and making practical what comes from outside. I.e. in order to be able to apply results of research in an effective way, teachers must to some extent be able to do research themselves. The preparation and conduction of clinical interviews about fresh themes should be a good introduction.

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DEVELOPMENTAL RESEARCH ON PAPER AND IN ACTION

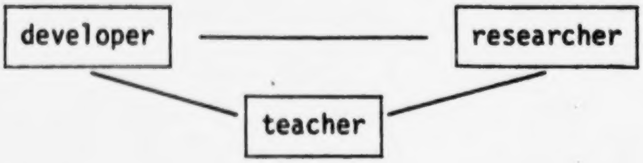
PIET VERSTAPPEN

STICHTING VOOR DE LEERPLANONTWIKKELING (SLO)
ENSCHEDE - THE NETHERLANDS

As indicated by the title, this paper is a contribution to the theme "Systematic co-operation between theory and practice in mathematics education". A sub-theme is theorization about the relationship between theory and practice, between didactical research in a broad sense and educational practice. Essentially, the theme is a specific case of the more general relationship between everyday and scientific knowledge, as Husserl, for instance, described in his book on the "crisis of the European sciences" which deals with the problems of the alienation of science from everyday life. I will leave to one side the products of the research, such as learning materials and curricula. Moreover, I will restrict myself mainly to the origin, essence, finality and activities belonging to the indicated relationship.

The activities of didactical research are of several kind such as investigation, development, comparison, evaluation and application. Many countries bring forward important work in this area. Not so The Netherlands. Not because we would not want to - on the contrary, I would submit - but we operate in a different manner. The structure differs, and particularly the position of Dutch curriculum development, which is unique in several aspects. In the first place, a great part of our curriculum development has been centered in one institute, which means that other aspects such as research, support, test development, pre- and in-service training have been located elsewhere. This separation undoubtedly has advantages, e.g. a better co-ordination between subjects is possible. However, there are disadvantages as well. It is clear that educational development is thus split up and is dependent on the co-operation with other curricular disciplines. Success stands or falls with the willingness and especially the possibility to co-operate, since time and means must be available in parallel. Secondly, our situation is unique because our work is not primarily aimed at the production of learning materials, but rather at more global working plans of the schools and at curricula at macro level. The government, who finances us for the greater part, considers the development of learning materials to be a task for publishers.

With respect to the SLO-project "Mathematics 12-16", reasonable co-operation has been established. For some time now we have experienced co-operation with several independent disciplines. It is therefore that our contribution to our common theme deals with an additional component. Namely, besides a relationship between research and practice, also a relationship between the components of research themselves. To keep matters from becoming too complex, yet dealing with the topic within the given time, I will restrict myself to the triangular relationship between



Thus, the theme has been narrowed down to the issue of co-operation between developers, researchers and teachers, but that limitation will do for the moment.

Before dealing with the issue more closely, some data about the case. Involved here are six young SLO developers who together form an enthusiastic team, three researchers from a department of the University of Utrecht, and some twelve teachers of whom three work at training institutions and the rest at five practice schools. The research group is financed half by the SLO and the teachers are paid by the SLO in part. Obviously, the development group constitutes the heart of the project, although - or rather because - the research group maintains a formally independent position.

The principal goal of the development group is to come to a school working plan i.e. an overall curriculum design for mathematics to 12-16 year-olds. The following are the starting points of the group:

- mathematics for all;
- directing mathematics more towards everyday life;
- the presentation of mathematics from contexts which correspond with the intuitive knowledge of the pupil;

- relating mathematics emphatically to the personal methodical approach;
- the right to learning, with insight for all;
- a working form which is flexible in principle, but particularly the effectivity of the heterogeneous table group, is a subject for testing and investigation;
- differences in progress and levels are accepted while subject matter and pace are similar for all.

The theoretical objectives of the research group (Intra-class Differentiation Mathematics Education 12-16) can be summarized as:

- contributing to the development and improvement of the learning materials;
- elucidating the concept "mathematics for all".

The aim is research for the benefit of curriculum development and a mathematical-didactical coloured theory of education, the latter a specific modality of mathematics education for 12-16. Another aim is to trace objects/means relationships that are employed in the development of the curriculum in order to derive a descriptive explication of the basic concepts of the curriculum.

Besides the theories already mentioned, all participants are confronted with theoretical views on their specialism. In his thesis, Terwel discerns 7 research approaches:

1. Evaluation research (USA)
2. Naturalistic research (USA, UK)
3. Action research (Western Europe)
4. Systematic education experiment (Eastern Europe)
5. Design theory/construction research (USA, Belgium, Holland)
6. Education developmental research (Holland)
7. Developmental research (Holland).

In our case, it concerns education developmental research which in short can be defined as an informal, incremental, developing approach with strong emphasis on improvement of educational practice and development of learning materials. The researcher fulfils many roles: developing, training, supporting, investigating. He is allowed to interfere and influence spontaneously and he plays an important part in the dissemination of results and the conviction of various groups. The developer's task is markedly creative. He designs new learning materials and tests these for their usefulness. He therefore works in the school. We do not, or rarely, try-out in laboratory settings. The developer actually gives the lessons or participates in them, in order to be able to observe the actual learning processes from nearby. He analyzes, improves and records his findings in documents which on the one hand contain plans for the arrangement of mathematics lessons, and on the other have an inspiring character for teachers, teacher-trainers and textbook authors. Rarely does the teacher explicitly confide his starting points and goals to paper. His inspiration lies most strongly in classroom practice.

On paper, all seems well arranged, Developers and researchers wrote project, planning and execution documents, stated their goals and starting points explicitly and attuned these. Together with the participating teachers and their school staffs, many consultations were held. Most of the documents were cognisable. All seemed to be well - you will notice that I use the past tense. It is not that relationships are poor, on the contrary, one gets to know, respect and understand the other better when working together, yet one becomes conscious of the existence of tensions - not between people, but between ideas, points of view and conceptions, although these cannot yet be made explicit.

Participants in education developmental processes are not yet really aware of this diversity in views, I believe. Theoretically, one agrees with one another, so it is argued. If in spite of this something goes amiss in practice, it is blamed on a shortcoming in communication. Let us elaborate on these aspects: "agree" (viewpoints), "go wrong" (problems) and "communication", by applying them to the three pairs in the triple relationship: developer - researcher, developer - teacher and researcher - teacher.

The relationship developer - researcher

The most important difference originates from the attitude towards theory. Firstly, there are several different views on theory which correspond with the philosophical view on man and society. It seems to me that the most objective form of theory is indicated by K. Popper when he describes theory as that which has been put down in documents, books and such belonging to a third world besides the physical and subjective world. Others share a more naturalistic conception of theory (Quines), "knowledge, mind, meaning and values are part of the same world, and are to be studied in the same empirical spirit that animates natural science". Realism on the other hand, views theory, language and method as the reflection of reality. I prefer the more scholastic view in which theory is considered as a modality, a way of being. In this case, being stands for real, realistic or reality, practice, existing, and such. The modality also encloses the subjective and is a product of the activity. Nature and mode are never objective and depend upon community and the individual. Being is a categorical construct; its context states in greater detail which area of being is at hand. For instance: being human, education research, biosphere.

In this view, theory or modality of being translates and explains an area of being with the aid of concepts, judgements and a system of proof. An essential difference now originates, since the translation can have a descriptive or a prescriptive character. In the first case the chosen position is neutral, in the latter it is normative. Theory then must be a standard; a guideline for practice. The difference all boils down to the difference in starting points and goals. With this we touch the essentials which greatly influence the relationship developer-researcher, because already earlier we pointed out that the curriculum is the precipitation of the developer's prescriptions while the description of the researcher has a theoretical elucidation in mind.

In a way the researcher allows practice to prevail more than the developer does. Theory arises and emanates from practice, while the developer wants to adapt practice to his theoretical view. If it does not correspond with the imagination or the starting points of the developer he then comes up with new tasks. The developer does not like to state his aims explicitly beforehand on the level of learning materials. This causes resentment on the part of the researcher who prefers evaluation of practice, i.e. comparison with accepted aims. However, the development of pre- and post-testing does emanate from the demands of the developer. Developers are not readily prepared to drop a once chosen idea, elaborated in learning materials. Naturally, they are willing to adapt the material, which often implies extension upon it. Researchers are farther removed from this material and therefore will sooner suggest the elimination of large parts of the package, or the combining of packages. The researcher wants the developer to define the differences in level, to test the work in heterogeneous groups. Is it really true that this didactical approach shows a communication between pupils about the tasks from different levels? The developer already seems satisfied when his material evokes any discussion at all. Whether this takes place at different levels, it is something which has a lower priority on his list.

Another problem concerns the difference in pace. In a way, the researcher's work is more fundamental, therefore requiring more time per package. His results display a time-lag. The developer has already tried out several versions of his own, often in several schools at the same time. He is satisfied. The results of research are just starting to come in when he - the developer - is already working on something new. In other words, the developer's work is broader while the researcher's is deeper. This difference in strategy is a serious handicap for the co-operation because the developer does not experience the positive input of the researcher during his engagement with the material.

In our case, good communication has been established. Both teams meet, under my chairmanship, every six weeks. Prof. H. Freudenthal also participates. Furthermore, one member of each team is responsible for communication for the time in between and both prepare the business for the plenary meeting which deals with learning materials, co-ordination, planning, exchange of experiences etc. Both groups operate in several practice schools of which one is common. It also helps that a member of the research team is originally a mathematician which allows us to speak on equal terms. Dissemination of the results occurs on a broader front where researchers meet with yet another audience. It is true that this led to a number of competency problems, especially in the beginning, concerning the authorship of modification. Let me summarize here with respect to viewpoints, problems, and communication.

<ins>Viewpoints</ins>:
- common
- different positions on theory

<ins>Problems</ins>:
- unequal pace
- being explicit on aims and levels
- differences in objectives

Communication:
- about learning materials
- about experiences with the children
- frequently and well established

The relationship developer - teacher

This pair also holds that a common view must exist from which to start. The teacher is dissatified with the current situation and is looking for something new. He expects new contents and methods to lead to a more harmonious form of education. The theoretical description appeals to him, although much of it escapes him. This means that the teacher has adopted an attitude of waiting in which he wants to see the ideas worked out on lesson level. Originally it was intended that the teacher would co-operate in the development of the ideas. However, this co-operation cannot be called successful. The ideas and products which were brought forward by the teachers fit the current curriculum while the aim was to design a new one. In fact, before long teachers restricted themselves to feedback on the presented concept-materials. Of course their comments were based on their judgements with respect to feasibility. Most often that meant underestimation. The developer often made several versions of the material before trying it out in the classroom. They often extended upon these versions, based on their findings, especially on recent experiences in the classroom with tests on the subject. They then modified their actual try-outs. The appendix is an example of this process. Note the slight input of the teachers. It is true that their attitude changed during the development process. Once they had observed that the children enjoyed these tasks and did reasonably well, they too became enthousiastic. We may say that the teacher team did not function as a development team, nevertheless they were useful in providing feedback and supplying protocols of their lessons.

The teachers regarded the adjustments made by the developer mostly as simplifications or eliminations of the more enjoyable. The ideas of the teachers were rarely used since they did not correspond with those of the developer. We summarize as follows concerning communication.

- The teachers speak much the same language or jargon as the developers, both use more of an everyday language. All of the developers were former teachers. As a variant on Shapere's "as we learn we learn how to learn", we can say "as we teach we learn how to teach". Strictly speaking there is no gap between the everyday school knowledge of the teacher and of the developer.

- There are numerous contacts, with peaks during the trial period.
- There is great openness between partners.
- Contact in writing does not work well, the manuals of the topics are read poorly.

Most of the problems stem from peripheral conditions of school education. Especially the underlying hidden curriculum which is manifested during testing, selecting and marking, makes demands of its own. Matters in which the developer is less interested. As I said before, teachers often first of all react by saying that certain learning materials are too difficult for their pupils and then they start working with the material in another way than it was intended. The teacher pays attention to the effects on his pupil and on his classes, while the developer is mainly interested in the final version that is ready for distribution on a wide scale. In other words the teacher looks at the local, the developer at the national situation.

The relationship researcher - teacher

We will deal with this relationship only briefly.

Viewpoints

The points of view on education are not really different, but barely made explicit and one is therefore not inclined to discuss this topic.

Communication

On the one hand less simple, because researchers use more of a jargon.

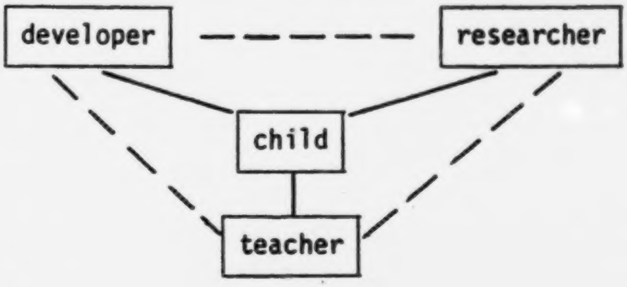
On the other hand there is more openness about materials. The developer is still more emotionally involved. One is sooner prepared to tell a researcher that a certain topic is wrong.

Problems

At the outset the participants are not frank with one another. Teachers are somewhat afraid of researchers. The teacher feels that he is being evaluated. Later there is more openness.

> The researcher hopes for the best possible similarity in the conditions at the schools, to make comparison possible. Teachers are not interested in this similarity. So it happened that a certain topic was found to be unsatisfactory. The teachers at the two different schools reacted very differently. There was no protest at one school at all but the work in heterogeneous groups was discontinued to treat the topic in the class as a whole. The other school dropped the topic and looked for other subject matter that was suitable for teaching in groups.

Let us recapitulate: The activities of didactical research are of various kind. In The Netherlands these activities are accommodated in different institutions. Besides an external relationship between theory/research and practice there is also an internal relationship in research itself. Specifically the relationship between developers and researchers. That relationship is manifested in classroom practice. The theme: Systematic Co-operation between Theory and Practice, has been narrowed down to the issue of co-operation between researchers and teachers. It then becomes evident that communication is not attuned to the differences in understanding or language but that there is a gap with the expectation concerning the usefulness of scientific knowledge. Teachers hide from theory but on the other hand are susceptible to practical learning material. Real communication takes place via the child, i.e. through practice. Theoretical starting points or aims do not penetrate practice, unless evolved from them. It is for that reason that we must extend our triangle.



Teachers, developers and researchers understand each other in the light of practice. In other words, communication takes place via the child in the classroom. There is another gap between developers and researchers which emanates from the opinions about theory. To developers theory is prescriptive and normative, therefore a guideline for practice, while for the researcher theory possesses a descriptive characteristic and need only reflect modality. Teachers are not concerned about theory, they work and think from everyday experience.

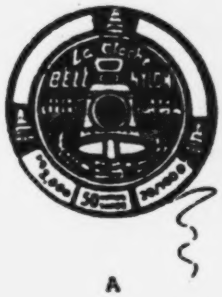
Appendix

In previous topics, pupils learned something about relationship and graphics in an intuitive manner. The following topic is intended to deal with graphs more exactly. In the first part the situation is described and explained. In the following the data are arranged and for the first time a relationship must be made explicit.

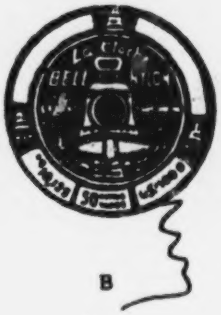
First version (in my lay-out)

Fishing

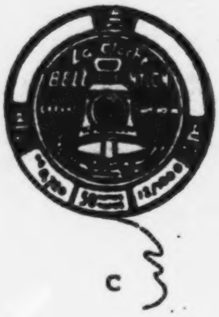
Fishing line is bought from the roll. You have a choice of several thicknesses of line. Different thicknesses have different tractive power. The thicker, the stronger. Below are some examples of the labels that can be found on the spools of line.



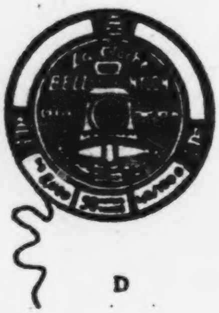
A



B



C



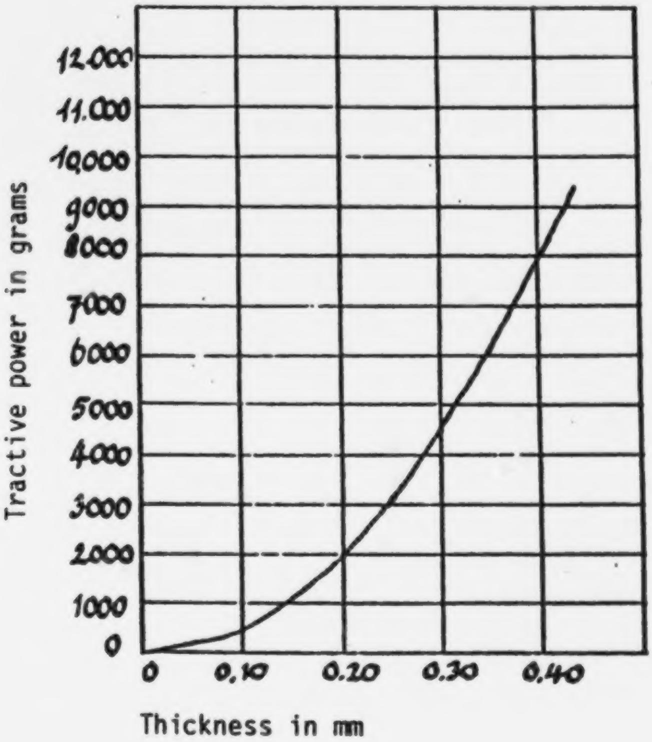
D

On spool B it says "kg 10,125". What could this mean? What does "20/100 0" mean?

Make a table in which the following data must be recorded:

line	thickness in mm	tractive power in g
A		
B		
C		
D		

Each thickness has a definite tractive power. Expressed in a graph this looks like:



A great deal can be read from this graph.

1. How heavy may a fish be if you are using a 0.45 mm line?
2. A mackerel fisherman mostly catches fish that weigh between ½ and 2 kg. How thick must his line be?
3. Try to find the tractive power of a 0.550 mm line from the graph.
4. Make a statement that is true: if I want a line that is twice as strong it will have to be .... as thick.
5. Do you have any idea of how much you can hang on a 1 mm line before it will break? How can you find out?

This version functioned reasonably well in the classroom. Yet certain modifications were suggested. Question 1 was changed because it was not clear. Thus, the pupils were inclined to look at the graph for 0.45 mm. The intention was more to aim at the upper limit of an interval at the maximum weight this line could hold. The formulation of question 4 was also changed. Perhaps the interchange of question and answer will give better results.

This is the second version

A great deal can be read from this graph. Try!

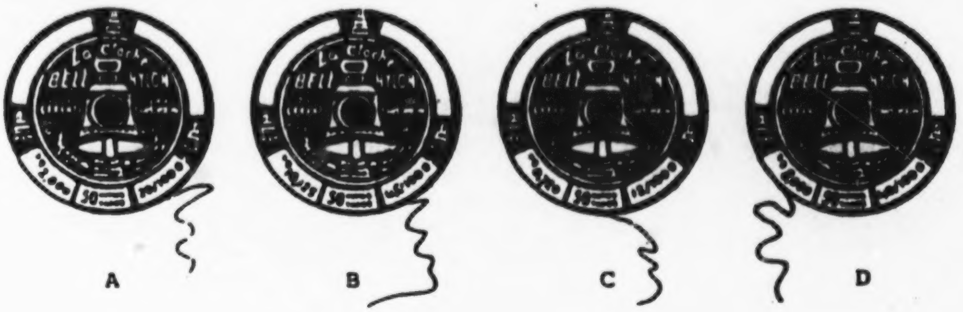
1. If you are using a 0.45 mm line, how much can the heaviest fish that you can catch with this line weigh?
2. Identical.
3. Identical.
4. Complete a true statement: if I want a line that is twice as thick, it must be .... as strong.
5. Identical.

The last question still did not work well. The relationship was not discovered and the table did not function as a useful mathematical tool. In a way this result was dissapointing bacause the developers had faith in the suitability of the context to allow the pupil to invent the quadratic relationship. Next, the developers had the idea of introducing a central problem within the context. One that was intended to arouse curiousity and motivation. They introduced Gemma who was wrestling with the problem: "Can you catch a 40 kg fish on a 1 mm line?". This problem seemed suitable for experiencing mathematical activities. How do you tackle such a problem? Which means can be used? Why do you accept one thing and reject another? In short, the objectives changed with the introduction of the new text. The figures were also adapted; "reality" was not yet quadratic.

At the end of the third version

"I saw someone catch a fish that weighed at least 40 kilo's", Gemma announced when she came home. "That is impossible, you would need a pole with a thick line just to get it out of the water", her father answered. "Not at all", Gemma said, "I don't know what kind of rod it was but the line was only 1 mm. That is what they said at least." Her father did not believe it. "According to me that is impossible. The fish must have weighed less. They were exaggerating, I'm sure. It is probably one of those strong stories fishermen tend to tell." Gemma has her doubts now. "Do you suppose it wasn't true?". But she saw for herself that the line was very thin. And it did take two of them to pull the big fish out of the water. They had said "40 kilo's on a 1 mm line". Perhaps she had misunderstood them.

What do you think, was Gemma right or not? Gemma is far from satisfied. Let us see what she is going to do. And of course, participate as well. "What different kinds of fishing line are there? How thick are they and how much can they hold?", Gemma wonders. She goes to a store that sells all kinds of fishing gear. They sell four kinds of line. These are the labels.



Take a look at the label on spool B. It gives some information about the kind of line on that spool. "kg 10,125" means that the line will break if you hang something on that weighs more than 10 kg and 125 grams. "45/100 0" means that the line is 45/100 mm thick. The sign 0 is an abbreviation of the concept diameter ... .

A few questions:

a. How thick is line C?

b. How many kilo's will it hold before it breaks? How many grams is that?

c. How many yards of line are there on the spool?

Back to Gemma.

I want to know how much a line of 1 mm can hold. They don't sell that here. And of course they wouldn't know either. But perhaps this information will help me anyway. She writes down the information on the labels on a piece of paper.

d. Record the information in a table. Write down the information about the thinnest line at the top and go on from there.

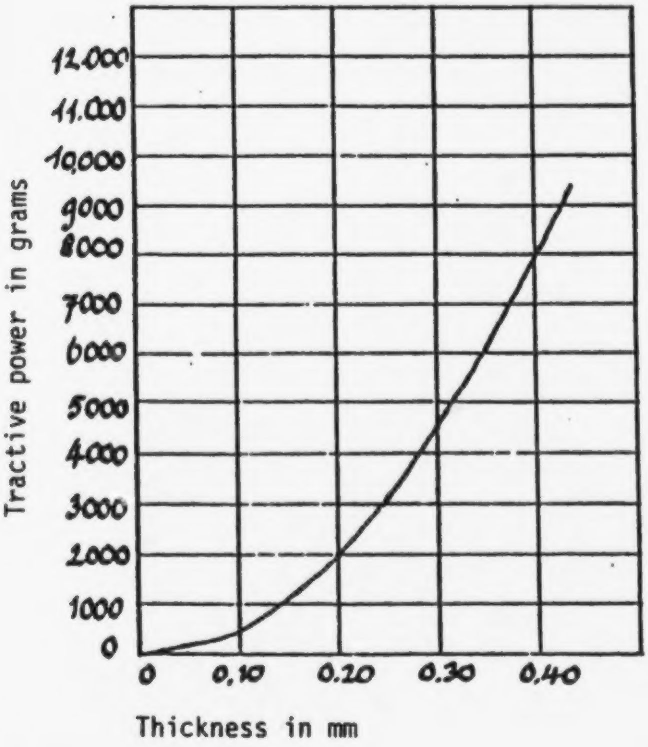
line	thickness in mm	tractive power in g
C	0.12	720

Using this information I should be able to predict something about other kinds of line, Gemma thinks. It looks like a line of 0.08 mm can hold exactly 16000 grams that is 16 kilo's. A 0.09 mm line will break at a little over 20 kilo's. Well, if that is correct, I could not have been right. Or do you suppose I'm not calculating it correctly?

e. How did Gemma figure this out?

f. Do you think her method is right? Why?

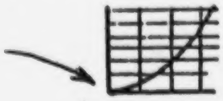
In the library Gemma looked up a book about fishing. In one of them she saw the following graph:



Oh boy, a graph that shows the strenght for each thickness that is handy. Let's see if it is the same as the line from the store.

g. Indicate the lines from the store in the graph with a dot. Write down the letters next to the dots.

h. Why does the graph begin at this point:



You can read quite a lot from this graph that could be of importance for Gemma's problem: whether a 1 mm line can hold 40 kg.

Try to find the following answers from the graph:

1. How thick must the line be for a mackerel fisherman? (between ½ and 2 kg)

2. At how many grams will a 0.50 mm line break?
A few pages back you saw that Gemma made some predictions about the tractive power of lines that were 0.80 and 0.90 mm. You have already given your opinion of her method. Now you also have the graph at your disposal. What is your answer to the following question: were Gemma's predictions too high or too low?

3. Would you dare to predict the breaking point of a 1 mm line?

Back to Gemma.

"Let's see", Gemma says, "a line of 0.10 mm has a breaking point of ...... grams. A line that is 0.20 mm has one of ...... grams. Line that is twice as thick, is ...... times as strong. What does that mean? Let's see for 0.15 and 0.3 mm. I think I've got it because I can figure out how much a 1 mm line will hold like this." After some time, Gemma starts beaming. Her conclusion: "I was right after all".

4. Try to find out what Gemma thought of and what she figured out. Fill in the missing numbers and find out whether Gemma is right or not. Write down your conclusions and calculations below.

WORKING WITH TEACHERS TO IMPLEMENT A PROBLEM SOLVING APPROACH WITH PUPILS AGED 9-13 YEARS

LEONE BURTON

MATHEMATICS SECTION
AVERY HILL COLLEGE - U.K.

Introduction

The view of problem solving which underpinned the project on which I wish to report was that of Polya:

> "If you have genuine *bona fide* experience of mathematical work on any level, elementary or advanced, there will be no doubt in your mind that, in mathematics, know-how is much more important than mere possession of knowledge .... What is know-how in mathematics? The ability to solve problems - not merely routine problems but problems requiring some degree of independence, judgement, originality, creativity." (Mathematical Discovery, Vol. 1., vii, Wiley, 1962).

Know-how develops from active involvement. Confidence grows not only because of success but also out of a sense that the action is under the control of the actor who is, to some degree, an independent decision-maker. When learners exercise control in this way, the pace, the intensity, the excitement and the atmosphere alter. So pupils were to be offered a challenge to which they had no ready-made reply, no algorithm, correctly remembered or not. But the purpose was not simply to provide an opportunity for them to be problem solvers in mathematics. It was felt to be of equal importance that they improve their ability to make choices by reflecting upon and assessing the processes which they used. The distinction was made between:

> *a problem class* in mathematics which has as its major objective facilitating students to find solutions to problems or exercises on a particular topic. This is a standard teaching format for providing experience and practice for newly learned techniques in the mathematical and physical sciences; and

> *a problem solving class* which has as its major objective the presentation, experience and consideration of those problem solving procedures which contribute to the process of uncovering a solution to a problem.

It was the latter format which was of interest to the project team.

The current situation in schools in the U.K. is that many teachers of mathematics are anxious and insecure. This is especially so in primary schools (pupils aged 5-11 years) where teachers' insecurities can often be traced back to their own experience of mathematics in school. Lack of confidence leads to rigid and stereotyped curricula and methods, heavy reliance on texts and insistence on uniformity. Questions are discouraged, alternative methods or solutions are threatening.

However, at the same time, teachers are most self-conscious about this situation and eager to apply their knowledge of pupils to a deepening awareness of what mathematical activity entails. Since problem solving demands creative curiosity, exploration, investigation of alternatives, comparison, discussion, and evaluation of results, teachers being asked to shift to a fundamentally different pedagogical style needed support in finding and maintaining a new and different role.

The project team hoped to establish an environment which encouraged the teacher as researcher in the classroom. However, it had to be recognized that little was known from research of how problem solving affected the learning, especially of younger children. Work which had been done with older children and students (Kantowski, M.G., in Krulik & Reys, Problem Solving in School Mathematics, 1980 NCTM Yearbook, Reston, 1980; and Schoenfeld, A., About heuristics: what they are and how we can teach students to use them, NCTM 1979) suggested that the processes used in solving mathematical problems are acquired slowly. The intention here was not so much to test the acquisition of processes as to establish that they already existed in young learners and that in encouraging their use, the learning climate in the mathematics classroom would change.

Pupils, too, have a restricted and stereotyped view of mathematics. The absurdity of pages of ticks gained by repeating and repeating an operation which is understood and can be used successfully is juxtaposed for some with the anxiety for others of pages of crosses, when failure to understand is reinforced. Worst of all, it was felt, was the lack of recognition of mathematical thinking skills already successfully part of the child's repertoire of behaviour. Acknowledgeging these, and encouraging children to use them consciously in class and to build upon them, would, it was hoped, help to turn mathematics from a task which for many was abstract and meaningless, to an exciting and creative experience. In particular, the following were to be emphasized:

1. Entry behaviours: Entry difficulty had been observed as much in closed, examination-type questions as in open, investigatory-style problems. Pupils find difficulty in familiarising themselves sufficiently well with the task and the information that has been supplied to be able to attack a problem with confidence. To facilitate entry, two behaviours were encouraged, exploring and representing.
2. Communicating: Written presentation of mathematics and its relationship to meaning and action is another area of persistent difficulty. Consequently, the emphasis on representation was to be matched by an equal emphasis on communication, the role of recording, when and how to use informal and formal means of presentation, discussion, etc.
3. Resolving: pupils were not only to be encouraged to check their solutions and look back for interesting and informative features, but the activity of solving problems was presented as being ongoing and cyclical so that one problem resolution leads to the development of new problems. Problem posing and resolving, it was felt, not only approximated more closely to experience, but would build pupils' confidence and change their attitudes to the learning of mathematics (see Brown & Walther, The Art of Problem Posing, Franklin Inst. Press 1983).

The project aimed, therefore, to affect both teachers and pupils. For teachers, a structure was to be offered which would support changes in pedagogical role through which, it was hoped, insight would be gained into pupils' mathematical experience. For pupils, an alternative experience of mathematics would, it was hoped, build confidence, reinforce problem solving skills and change perception of mathematics. The specific objectives were:

1. to develop a reference tool which displayed the breakdown of skills and procedures of mathematical problem solving;

2. to develop a structured programme of problem solving assignments for the teaching of problem solving skills and procedures across the age range 9-13 years.

Implementation

This report will concentrate on the implementation of the second objective for which the project team and the contributing teachers worked together.

Teachers in five diverse areas of England were invited to participate in the project. Those who volunteered and joined the study came from London, the Home countries, the North East, the North West, and East Anglia, and taught in urban and rural schools, organized as primary, secondary, or middle. In all, 27 classes of approximately 810 children were involved. For each class, a similar control class was pre- and post-tested.

Two requirements were laid down by the project team for teachers who were collaborating. One was that they devote a minimum of one hour per week for one term to the problem solving. The second was that during that hour, they perceived their role as a resource rather than as an informant and to this end, classes were to be encouraged:

(a) to work in small groups with whole class work restricted to initiating an approach or sharing a particular aspect;

(b) to act independently of the teacher exploring, discussing, and deciding themselves with no time pressures, and no expected conformity of outcome.

To support the teachers in making this shift, a bank of thirty problems for the pupils, with supporting information for the teachers, was developed by the project team. In addition, display materials were devised to reinforce the approach. All the relevant materials were circulated in advance so that at the first meeting of the teachers and project team, time could be spent in discussion rather than in familiarisation.

In each area, one day was allocated at the end of the term preceding the start of the project for a meeting between teachers and project members. At this meeting, the morning session began with small groups working on a problem from the pack so that there was a basis of shared experience from which the philosophy and approach of the project could be derived. The afternoon session was used to discuss the project materials, classroom organization and administration. Stress was laid upon the changed role of the teacher, the necessity of using concrete representations, the avoidance of a single solution approach, and the distinction between the use of informal and formal methods of recording. The communal problem solving produced a good atmosphere which was not intimidatory and allowed a variety of methods and a wide range of skills and procedures to be utilized and discussed. It also established a model for use in their own classrooms. The teachers' reaction to the day ranged from 'very useful' to 'essential' and they commonly expressed the view that more time spent in this way would have been helpful.

Teachers were encouraged to withdraw any problems from the pack that they felt were unsuitable for their class and for this reason considerable information on each problem was included in the teacher's version. Sample resolutions, for example, provided enough backup to instill confidence and ideas for extension and interventions were included. Teachers commented on the usefulness of these in such remarks as "the guidance on interventions was helpful in controlling progress, particularly until I became familiar with the project". At all times, responsibility for the class rested with the teacher, although members of the research team regularly visited all classes. However, it was not unusual to visit a class of 10-11 year old pupils in the morning and observe them working at a particular problem only to have the teacher of a secondary class of 12-13 year olds, in the afternoon, say that the same problem had been withdrawn because of difficulty. Responsibility for the work, as far as practicable, rested with the pupils. There was free choice of problems, pupils being asked to record when a problem was selected and when returned. In this way, it was possible to assess problem popularity, frequency of change, time spent. No stigma was attached to choosing and then rejecting almost immediately. It was found that problems achieved class reputations according to the experience pf the pupils and that once a problem was circulating in a class, it would be widely attempted whereas a problem that was neglected in the early sessions tended to remain neglected. Pupils freely commented to one another on the reputation of a problem but were most circumspect in not commenting on their resolutions. Thus, while saying "That's a good one", or "That one is fun", they were not observed saying something like "Do it this way" or "Try that".

Results

Results were obtained in three ways. All pupils in experimental and control classes were pre- and post-tested. Ten percent of the pupils in each experimental class were interviewed on their attitude to the project work, to mathematics, to the relationship between the two. These pupils were also asked their views about the difficulties involved in problem solving and their feelings about the term's activities. Every teacher on the project was interviewed as well as being asked to complete a questionnaire requesting responses to project materials, to the pupils' attitudes and progress, and to the relationship between mathematics and problem solving.

The Pupils

A substantial shift was displayed by the end of the term, from early dependency on the teacher for guidance and assistance to a flexible and independent approach. Entry behaviours initially were constrained, especially by lack of attention to reading, but increased confidence and less demand for teacher intervention was evident as the term progressed. Pupil material successfully supported this in exhorting, for example, "Read it, really read it", or "Make a start, write or draw something". The value of representing a problem was quickly appreciated and children began to exercise discrimination in the choice of suitable materials. Communicating was more difficult although pupils were highly inventive in the methods they used to communicate their results. As far as the pupils were concerned, the least effective process was that of extending the resolution of a problem into new and different problems. However, this process is new to teachers as well as pupils and classroom expectations tend to operate against it. Most school work is aimed at reaching a correct answer and pupils found it strange to be expected to continue further with a piece of work when they considered they had arrived at a successful conclusion. For teachers, too, such extensions can pose a considerable threat as, often, pupils raise questions on which the teacher has not pondered and for which she might have no obvious strategy. It requires considerable teacher sophistication to be ready to accept such a situation. Pupils' enjoyment of the work of the project was evident and was supported by such comments as "Gave you something to think about" and "Gets your brain working and really stretches your mind".

The Teachers

Problem solving was not only accepted as an aspect of mathematics which complemented the more usual curriculum but all the teachers acknowledged its value. Although at the beginning of the term the teachers were willing to join in what they saw as an experiment, by the end of the term they commented on its necessary inclusion as part of the education of their pupils. In answering the question "How do you view the place of problem solving in school mathematics?", they wrote replies such as: "Essential, and a confidence booster to some of those to whom formal work is difficult or laborious". - "It gives meaning to the basic rules the children learn. They get a positive attitude towards novel problems". - "I should like to include this at some time in each child's education".

The project materials had been given to the teachers and they were asked if they proposed to continue to use them after the experimental period. All replied that they would and later follow-up confirmed this.

In responding to the question as to the ways in which the project had benefited their pupils, teachers were enthusiastic about the increased use of materials and apparatus, the growing care taken in reading and interpreting, and the value to be gained in looking at problems in different ways. One comment was "the method used is often as important as, and certainly more useful than, the final result". Teachers observed improvement in initiative and persistence in individuals and in discussion, argument and reasoning in groups. Project emphasis on "being systematic" was supported and the teacher of one class reported that "maths isn't just sums" and that the pupils were transferring their problem solving skills to other areas. In general, results confirmed that a programme of this kind can be implemented by non-specialist teachers, they can adjust to the role of resource rather than pedagogue, and the feasibility of implementing such a programme is not affected by type of school, location or ability range of the pupils. Most importantly, however, the teachers participating in the project commented on how much they had learned about the teaching and learning of mathematics. The project team felt that no amount of formal in-service work could have provided the experience on which so many of the collaborating teachers commented because they retained overall direction and responsibility for their class. Nonetheless, there was considerable support for the teachers both in the terms of personnel and materials. In the longer term, where this is not feasible, self-help groups of teachers possibly associated with teachers' centres would have to provide sufficient backup. The initial contact with the project and its materials was crucial and required extending. Experience suggests that it is facile to expect teachers to undertake the radical shift in style that work of this kind demands on their own, even where materials are available. This would indicate that either such developments should be centrally supported or that they should be undertaken on a school basis so that teachers can share their experiences and difficulties.

For the future, research interest centres on the implications of this work for teacher expectations of pupils and for pupils' self image in respect of mathematics. Some evidence over time of these implications being realized would be most powerful in respect of developments in mathematics education. In particular, it is important to establish the existence in young children of comprehensive mathematical thinking processes and to look for ways in which confidence can be built upon the recognition and use of these. During this project, such processes as specializing were recognized and utilized. So was hypothesis creation and testing. Convincing processes which depend upon communication were less successfully demonstrated but are, to a large degree, dependent upon classroom climate for their generation. This is equally true of the processes relevant to problem re-view and extension.

Some pupils acknowledged the applicability of the processes of problem solving outside the mathematics class. A more vigorous attempt to display this and make links across subjects would be valuable.

Most of all, the form in which the theoretical background to the project was presented was too involved to be easily available to a busy teacher. The team was left with the strong feeling that, for the future, two signposts were vital, one pointing the way towards a developmental approach to the recognition and use of problem solving processes and the other towards simplicity. A straightforward and clearly demarcated structure indicating how and in what combinations problem solving processes are recognized and utilized would make an important difference to the enthusiasm for this work demonstrated by teachers.

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RESEARCH AND TEACHER TRAINING: CO-OPERATIVE DEVELOPMENT OF TEACHING MATERIALS FOR STOCHASTICS IN GRADES 5/6

GERD VON HARTEN

INSTITUT FÜR DIDAKTIK DER MATHEMATIK
UNIVERSITÄT BIELEFELD - F.R.G.

0. Introduction

Curriculum development with teachers and in-service teacher education were recommended as mutually dependent innovation strategies in Western Germany in 1974. This can be interpreted as a reaction to the first stage of the curriculum reform which had led to some negative experience. The crucial difference from the former stage of the reform was that teachers were meant to participate in the development, and that activities were to be closely linked to in-service training. In the course of this process, a lot of projects working with different organizational conceptions to connect curriculum development and in-service training were established and financed.

These curriculum-projects in the Federal Republic have come to more or less negative conclusions with regard to the link between curriculum development and in-service training, and to the possibilities of developing material together with teachers. This is said to be due to the fact that it is impossible to link the development of material to the analysis of concrete classroom problems. This is very evident in one project which, in the course of its work, split into two groups: the first communicating the products of its work to teachers outside, the second focussing on concrete classroom problems. It is justified to assume therefore that reforming the curriculum is not the first thing in the mind of all teachers, but rather problems of everyday life in school such as discipline problems or the achievement of particular pupils. To conclude from these facts that it is impossible to integrate both these two elements - subject matter and social conditions - would seem to be hasty. Such a conclusion assumes that the subject matter and the social conditions of instruction are independent. To overcome this separation, however, is exactly one important thing in developing a conception of an independent school mathematics.

One thing we can learn from all these projects is that co-operation of researchers and teachers is highly difficult and that there are no simple solutions. Neither can the teacher be seen as a mere obstacle in the path of innovation, nor can the researcher's contribution be confined to theoretical conceptualization, implementation being left to the teacher. Theory, research, and practice belong together, but there is no direct and pragmatic way of bringing research and theory into schools. We need theoretical tools, tools which mediate between theory and practice, while keeping the functions of the researcher and teacher separate. Before describing such a tool, which we named "system of tasks"[1], the prerequisites of our project shall first be briefly described.

1. On the background of the project

Probability and statistics have just been integrated in the syllabus for the secondary I level of comprehensive schools in Western Germany, and educating and preparing the teachers are difficult obstacles in the establishment of this subject.

The goal of the project was not only to establish a new topic in the school curriculum, but probability and statictics were to be a novel type of applied school mathematics. The work on two levels, theoretical models and real random situations, was meant to develop applications and theory simultaneously, so that they explain each other. I shall not dwell on this here, since the theoretical background has been described by H. Steinbring (1984).

However, I should like to illustrate the strong contrast between this conception of stochastics and traditional mathematics teaching in the Federal Republic by the following comment made by a teacher in the classroom, where the students have been doing some experiments with dice:

> "Yes, if the probability to throw a sum of pips of seven with two dice is one sixth, then the sixth part of 1200 throws should show this result. Now we have a bit more than 1200, so how big shall the number of expected cases for this event be? Well? ...

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1) The German term "Aufgabensystem" is difficult to translate. "Aufgabe" lies somewhere between task, problem and activity.

> Yes, a bit more than 200, and what do we have? 171! I am not satisfied with this result; I am under the impression that some of you did not work exactly enough. When someone of you says, I have 81 sevens in 120 throws, there is something wrong, and I do hope that this result did not enter into our result. ... I have the impression that not all of you worked exactly enough ...". (From a lesson about the introduction of probability in grade 6.)

The challenging question could be: Is this still mathematics teaching, if doubts can arise as to the correctness of results, and if the teacher is unable to explain this definitely?

This shows that stochastics, in this term, requires the teacher to show new and extraordinary skills.

2. "Systems of tasks"

The project's objective was, on the one hand, to obtain insights into the form texts for teachers should have, and to develop teaching helps for the 5/6 stochastics curriculum of the comprehensive schools in North-Rhine-Westphalia on the other. The development was to involve active co-operation of the teachers. Our starting point was to prepare a first version of teacher material. This material contained the following two elements:

I A didactical analysis describing the didactical intentions of the lesson sequence and giving an overview of concepts and definitions for stochastics on the secondary I level.

II A system of tasks for stochastics in grade 5/6.

Placing tasks in the center was a move based on empirical studies in the Federal Republic (see e.g. Bromme, 1981) which had established that teachers plan their lessons mainly by means of tasks. The tasks were to enable teachers unfamiliar with stochastics to develop an idea of the mathematical frame of stochastics in secondary I level together with an idea of their actual teaching of stochastics in the classroom. For this purpose, the systems of tasks had to implement certain educational leading ideas while at the same time remaining open for different individual teaching styles in order to provide access to this new subject for different teachers.

The term "system of tasks" is a theoretical term developed at the IDM. Mathematical tasks which make sense never occur alone. They are always embedded in a broader problem context, which gives them meaning and explains them to a certain degree. This makes the difference between serious mathematical tasks and puzzles, riddles, mindteasers, etc. The context of a task in instruction mostly consists of other tasks.

An optimal form of putting tasks together in instruction is neither a battery of some which vary only in the numbers used, nor a colourful mixture of tasks which are only related by the same topic area. A substantial concept of "systems of tasks" which ought to be efficient, both theoretically and practically, will always be faced with the following problem:

On the one hand, there are ideal requirements, e.g. of mathematics or systems' theory, on the other hand there are practical conditions which require that such an ideal system be changed. The most important thing is to find a relational structure for related tasks, to use a metaphor, to interpret tasks as variables. The possibilities of varying these variables are situated *not* only in the mathematical area but also in the pedagogical, representational one or in classroom management.

To apply this concept to stochastics, I will give some practical rules which led us in the construction of "systems of tasks" for probability and statistics in grade 5/6.

(1) Systems of tasks in stochastics should contain concrete experiments and simulations. The activities thus required are addressed to different groups, i.e. individual pupils, a group of pupils, or the whole class. The system is linked by the common problem; the task may contain different requirements for individual pupils or groups. Organizing and co-ordinating experiments, simulations or polls together is to advance the connection of the task in the classroom towards a common goal.

(2) Systems of tasks in stochastics should contain decision problems. The constraint of having to make a decision, of being in some sense personally involved in a situation, is an important motivation for pupils.

(3) Systems of tasks in stochastics should contain different random generators or phenomena. The requirement to compare these mechanisms or phenomena motivates pupils to study similarities in the shape of variations or differences in the shape of changes of models.

(4) An important didactical principle of systems of tasks in stochastics is parallel work on an experimental and theoretical level.

(5) The tasks of a system are basically offers to the teacher. Each task should in some sense be a subsystem reflecting the system as a whole. There are various possible paths through a system. With regard to pupils' knowledge and skills, the teacher should choose the initial task and propose appropriate forms of work.

The experience gained by the project has confirmed that this form of material is useful. Tasks and their presentation in the classroom are, for mathematics teachers, a composite of mathematical, pedagogical, organizational, and social aspects. In contrast to mere task batteries, systems of tasks allow for flexible use in the classroom.

3. The development of the project

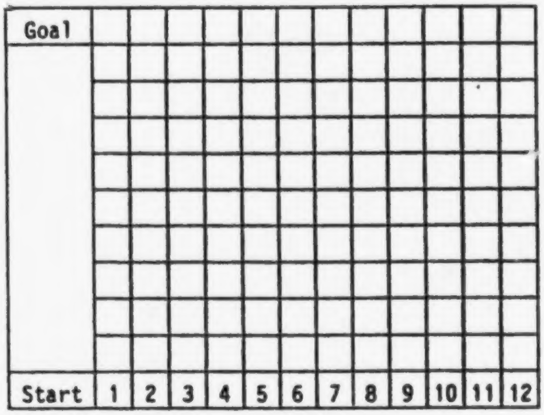
The first version of the teacher material was presented to teachers at five different comprehensive schools in North-Rhine-Westphalia. At these schools, all teachers in grade 5/6 (some in 7) used this material in their lessons. 26 of these teachers were closely observed by the project.

Subsequent to the teaching course, there were in-service teacher activities for teachers of comprehensive schools in North-Rhine-Westphalia in which most of the teachers co-operating with the project participated. These meetings served on the one hand to discuss the experience with the first version of the teacher material, and to improve the teaching practice of the participants on the other. (To establish a link between mathematical subject matter and practical teaching experience, groups of teachers discussed lesson planning with the help of the systems of tasks and lesson transcripts recorded during the first stage of the project.)

The final result of the project is at present a revision of the first version of the teaching material based on the suggestions of the teachers. An example will illustrate the type of suggestions which are included in the revised version.

A task belonging to the first version

This is a board for a game. To play the game you need two dice.



Choose a number from 1 to 12 and put your playing piece on the starting field with your number. Now two dice are thrown and the pips are added. Is this sum your chosen number, then your playing piece moves one field. Whoever reaches the goal first, has won.

(a) Note for each throw the sum of pips in the following table:

Throwing two dice

Sum of pips	Number of throw with this sum
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
More than 12	

Total number of throws: ☐

(b) Would you choose the 1 in this game? - Explain your answer!

(c) Which number or numbers are in your opinion good choices? - Explain your answer!

The result of the co-operation with the teachers is seen in the following hints, which are included in the revised version.

Hints for the teachers to this task

Pupils' previous knowledge

Special previous knowledge is not necessary. Knowledge on graphical representations is useful.

Concepts involved

This task also serves as an introduction to "randomness and strategy". It introduces a more difficult random mechanism, the throwing of two dice. A more detailed analysis of this mechanism could be done with task 3. [Another task belonging to the system.]

Methodological hints

This game should be played in small groups or by two pupils. Problems in understanding the rules of the game are rare, because most pupils know this kind of games. It is possible to give the pupils in the groups different duties: to throw the dice, to note the result, etc.

Special pedagogical and technical notes

The table of task 2(a) should be kept by the pupils. It is the first example of a distribution of results which are not equally probable. Other forms of representation can be introduced as column diagrams, histograms, and bar charts. A comparison with the table of task 1 (if this task has been dealt with in the lesson) could be useful. If task 3 follows this task. there should be a decision whether dice of different colours should be used in the game. Relative to pros and cons of such a decision see teacher hints to task 3.

Duration

This task can normally be carried out in one lesson. Experiences with work in groups and working sheets are:

5-10 min	Explaining the game and distributing the working sheets
20 min	Working in groups on task (a) to (c)
10 min	Discussion of results with the whole class.

How long pupils work in groups depends on how often they should play the game. If a pupil chooses the favourable number 7, he/she needs on average 60 throws to reach the goal, and deviations of more than 20 throws are rather improbable. In groups of four pupils mostly one of the numbers between 5 and 9 is chosen (with these numbers you need on the average 90 throws). One can assume that approximately 100 throws are needed per game.

Exemplary variations of the task

a) There are dice in the shape of Dodekaeders, with numbers from 1 to 12. If you have such a die, you can play the game with it and with two dice. This could give rise to interesting comparisons.

b) You can use a board with starting numbers from 1 to 18. Three dice are thrown and the pips are added. This game is much more complex for pupils.

This revised version is, however, not the last version. It was never intended to produce a final version. The material is to be a working book for teachers which is going to be altered and improved by teaching experience. The procedure might be called "independent evaluation" by individual teachers which serves to continuously improve the latters' teaching.

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L'ENSEIGNEMENT DE L'ÉNUMÉRATION:

ÉTUDE DE DEUX PROBLÈMES PRATIQUES ET FONDAMENTAUX DANS LE CADRE DE LA THÉORIE DES SITUATIONS ET DU CONTRAT DIDACTIQUE

GUY BROUSSEAU

IREM DE BORDEAUX - FRANCE

1. Introduction et résumé

Cet article a pour objet d'indiquer diverses formes d'intervention de la théorie des situations dans l'étude de problèmes pratiques d'enseignement. Ces exemples sont construits en utilisant diverses recherches dont certaines ne sont pas achevées, d'autres étant déjà anciennes. Il ne s'agit donc pas ici d'un compte rendu classique mais d'un exposé de questions et de méthodes.

Nous allons tout d'abord relever toute une famille de difficultés des élèves de différents niveaux à propos de différentes connaissances mathématiques, difficultés bien connues que l'enseignement ne résoud pas bien.

La construction de ce que nous appelons les situations fondamentales associées à ces différentes connaissances fait apparaître une condition commune capable de rendre compte des difficultés observées.

Cette première étape permet donc de formuler des hypothèses sur les causes des difficultés des élèves. Ces hypothèses peuvent être étudiées experimentalement par différentes méthodes classiques de la psychologie cognitive mais aussi par des méthodes directes, tirées de la théorie des situations elle-même.

Nous proposerons alors l'explication qui apparait la plus vraisemblable suivant les résultats que nous connaissons a ce jour.

Il apparaît alors qu'il est possible de la rattacher à une connaissance mathématique, ce qui permet d'utiliser a nouveau la théorie des situations, mais cette fois pour produire et étudier des séquences d'enseignement de cette connaissance, susceptibles de résoudre les problèmes d'apprentissage des élèves.

Les séquences d'enseignement produites "en application" de la théorie des situations sont généralement complexes et d'une conduite différente des lecons classiques. De sorte qu'ayant eventuellement résolu un premier problème d'enseignement on en trouve un second dans la difficulté a communiquer cette solution à ceux qui doivent l'utiliser. La théorie des situations indique comment et sous quelles conditions les micro-ordinateurs ouvrent des possibilités entièrement novelles dans ce domaine.

Nous ne pourrons pas dans le cadre de cette communication assez courte donner un aperçu de la théorie des situation elle-même mais nous évoquerons ce qu'on en attend dans le problème évoqué et nous résumerons la méthode d'analyse.

2. Étude de quelques difficultés d'enseignement

2.1 A l'université il est fréquent de voir des étudiants attendre la correction d'un exercise de combinatoire pour être surs que leur réponse est correcte. Alors que dans d'autre secteurs des mathématiques l'invention de la solution du problème est plus difficile pour eux que l'établissement de sa validité, lorsqu'il s'agit de dénombrements, curieusement les difficultés s'inversent. Certains étudiants ne savent pas comment se convaincre qu'ils ont bien compté tous les éléments et qu'ils ne les ont compté qu'une seule fois. On observe l'apparitions de méthodes de reconnaissance de cas - par exemple pour distinguer les arrangements des combinaisons - fondées sur des caractéristiques formelles non pertinentes (ils disent par example "c'est ordonné donc ce ne sont pas des combinaisons" alors que l'ordre dont ils tiennent compte est celui utilisé pour décrire un des objets a compter ...).

D'autres utilisent tout a fait au hasard les formules qu'ils ont apprises.

2.2 Dans l'enseignement secondaire les dénombrement posent aussi des problèmes difficiles aux élèves: ecrire tous les termes d'un produit de polynomes en troisième année ou faire la liste de tous les élément d'un ensemble qui ont une certaine propriété en première année ne va pas sans mal parfois même pour de bons élèves.

2.3 A l'école élémentaire certains élèves n'arrivent pas a savoir quelle opération il convient de faire dans les problèmes même si les manipulations évoquées leur sont familières et s'il est clair qu'ils les concoivent parfaitement. Les maîtres disent alors qu'ils n'ont pas le "sens des opérations" et essaient de le leur inculquer par la présentation répétées de problèmes que l'élève doit rapprocher des problèmes types. L'usage des caractères formels et des heuristiques douteuses est très répandu. Par exemple le maître justifiera une addition en soulignant dans l'énonce le mot "ajoute". Puis dans un autre problème ou le mot "ajoute" est encore utilisé, le maître modélisera la situation avec des petits nombres parceque la solution exige une soustraction mais si c'est d'une division qu'il s'agit il utilisera une autre métaphore.

2.4 En première année d'école primaire on peut observer des enfants qui "savent compter" mais qui ne mettent pas en oeuvre ce savoir pour effectuer des taches qu'ils comprennent bien pourtant et qui requierent le comptage: Ils ne comptent qu'en réponse à une demande sociale bien précise et n'ont aucune idée de ce à quoi cette tache sert ni ce qu'elle permet de maitriser.

3. Comparaison de ces difficultés

3.1 Quoiqu'assez différents ces divers problèmes peuvent être rapprochés intuitivement, il s'agit chaque fois de dénombrer mentalement un ensemble qui n'est pas concretement présent ou réalisé. Pour préciser ces ressemblances et les analyser il faut faire correspondre a chaque exemple, a chaque connaissance sa situation fondamentale. C'est un jeu tel que la meilleure stratégie implique l'utilisation de la connaissance visée mais qui peut être investi sans que cette connaissance soit nécessaire: l'élève peut comprendre les règles et commencer à jouer ou même gagner *sans* utiliser la connaissance qui commande la mise en place de la stratégie optimale. Dans le meilleur des cas la situation fondamentale permet la construction de la connaissance par l'élève lui-même au cours d'une suite de parties au cours desquelles il prend progressivement le contrôle de jeu. Cela implique naturellement que les enjeux soient connus sinon effectuifs ...

3.2 Ainsi on peut se demander par exemple quelle est la *situation fondamentale* de l'addition de naturels.

Il s'agira pour le joueur de produire le cardinal d'un ensemble alors qu'il ne peut en contrôler effectivement à chaque moment (pour quelque raison que ce soit) qu'une partie. Par exemple il ne sait pas compter des nombres plus grande que 5, mais il dit "il y a cinq et puis cinq et puis deux". Ou encore l'élève doit pour pouvoir les compter appeler certaines catégories d'objets, une a la fois: la stratégie optimale consiste a constituer une partition de l'ensemble puis a s'assurer du nombre d'objets dans chaque classe, en s'assurant que l'on compte bien toutes les classes.

Formellement le jeu doit permettre a l'élève de vérifier personellement sa reponse. Par exemple la reponse proposée est matériellement réalisée puis une association un a un permet de constater l'égalité.

De plus si l'on veut que l'élève *exprime* la somme et ne se contente pas de l'utiliser, il faudra ménager une communication ou l'émetteur pourra "interroger" la collection a compter et ou le récepteur devra réaliser la collection équipotante sans voir le modèle. S'il s'agit de prouver le bon usage ou une propriété de la somme, ou de débattre d'une méthode de calcul, la théorie des situation exige d'autre types de conditions.

La comparaison des diverses situations correspondant aux problèmes ci dessus (et que nous ne pouvons exposer ici) fait apparaître comme fondamental le contrôle mental par l'élève de *l'énumeration* d'un ensemble.

Par exemple on peut observer de grandes difficultés chez des élèves du début du secondaire a mettre en oeuvre le contrôle de l'exhaustivité même pour des ensemble finis de très petit cardinal au cours de raisonnements très simples.

3.3 Une énumération d'un ensemble est une injection de cet ensemble sur une section commencante de N. Ce serait donc la possibilité de produire et de contrôler la production effective de telles énumérations qui ferait défaut aux élèves en difficultés sur les questions évoquées plus haut. Il est clair qu'il faut prendre le mot "contrôle" dans un sens fort, et que le contrôle mental va exiger la mise en oeuvre *au service* de l'énumération, des connaissances logiques de l'enfant.

Nous donnons en annexe de cet article le début de la description formelle de la situation fondamentale de l'énumération, évoquée dans une réalisation sur micro ordinateur.

4. Étude du rôle de la connaissance de *l'énumération* dans ces difficultés

4.1 Remarquons tout d'abord que l'énumération est une notion qui n'a pas de statut culturel en mathématique, même si on peut la rattacher à des concepts et à des théories très importants elle n'est nulle part traitée comme un objet de savoir. Est ce pour cela qu'elle a échappé aux professeurs en tant qu'objet d'enseignement?

Il est facile de constater qu'elle n'est jamais enseignée sous aucune forme après la première année de l'école primaire même implicitement et que même la elle n'intervient effectivement que dans le dénombrement de toutes petites collections (généralement moins de 20) au dela le contrôle effectif de la collection par le maître lui-même devient presque impossible dans les conditions d'une classe.

4.2 Ainsi l'hypothèse suggérée par nôtre analyse des situations serait la suivante:

(1) Les élèves doivent passer très brutalement d'un contrôle perceptif de l'énumération de petites collections d'objets petits déplacables mais immobiles a un contrôle complètement mental et verbal d'ensembles quelconques.

(2) Les professeurs ne disposent pas de situations d'enseignement qui leur permettraient de faire développer par leurs élèves des savoirs faires des anticipations des stratégies dans des rapports expérimentaux avec le milieu. Ils ne disposent pas non plus de ce fait de moyens de négocier avec eux des contrats didactiques raisonnables à propos de ce questions ni même d'évoquer avec eux ce genre de difficultés.

(3) Ces conditions seraient une cause principale des difficultés observées.

4.3 Il ne manque pas de moyens d'étudier ces hypothèses. La méthode classique consiste a rechercher la preuve que ces difficultés sont bien dues a cette cause dans l'examen du comportement d'un certain nombre d'élèves d'abord sur des questions scolaires du genre de celle que nous avons relevées plus haut puis dans des expériences diverses d'énumération. Il s'agirait alors de s'assurer que ce sont les mêmes élèves qui ont des difficultés avec les différentes situations relevant de la même connaissance. Plus finement des observations cliniques pourraient mettre en évidence des filiations des hiérarchies ou au contraire des différences.

Il faut bien constater que, surtout pour des questions de didactique qui concernent des notions aussi précises et dispersées dans le temps ces moyens sont difficiles en mettre en oeuvre pour des chances assez faibles à priori d'obtenir des réponses très nettes.

L'analyse de textes, l'observation de classes peuvent aussi apporter beaucoup d'informations précieuses.

Mais que ce soit pour le réalisation des expériences du premier type, pour interpreter les comportements des élèves ou comme paradygme des problèmes et situations dans le deuxième il faudra réaliser des situations didactiques aux propriétés reconnues à priori. C'est justement ce que la théorie des situations produit le mieux.

Alors pourquoi ne pas essayer d'obtenir une preuve directe par l'étude de l'effet sur des élèves des différents types de situation d'énumération?

5. Les difficultés de la communication aux maîtres de nouvelles situations didactiques

Supposons que nous arrivions a démontrer par quelques moyens les hypothèses énoncées ci dessus. Les situations produites par l'ingénierie didactique appuyée sur la théorie des situations présentent généralement certaines caractéristiques:

- Pour permettre l'adaptation des élèves, elles doivent les conduire à prendre de nombreuses décisions, à en voir les conséquences, à les modifier etc.
- Pour permettre le fonctionnement du savoir, elles doivent autoriser diverses stratégies, correspondant aux différents points de vue, pertinents ou non, que les enfants peuvent avoir sur le sujet et répondre à ces comportements par des réactions appropriées.
- Irrémédiablement elles doivent exiger de l'élève un projet personnel et des relations sociales variées: communications, débats ou négociations avec d'autres élèves, avec le maître, etc. ...

Les situations qui satisfont ces conditions sont donc très complexes et difficiles à gérer. On les trouve rarement dans l'enseignement classique où le maître a généralement en charge de provoquer, de recevoir, de corriger et d'interprêter toutes les réponses significatives de chacun de ses élèves. Celles que la didactique a pu produire et qui sont gérables parce qu'elles assurent des autocorrections suffisantes sont le plus souvent encore plus difficiles à communiquer à un maître. De plus chaque savoir est caractérisé par toute une famille de situations obtenues en faisant varier certaines conditions qui ont une influence sur le savoir produit. Ces situations très nombreuses et chacune très complexe, forment une famille elle-même subtilement structurée de sorte qu'il est bien difficile de les connaître, de les identifier, de les reproduire et de les utiliser facilement en leur conservant leurs propriétés.

L'arrivée des micro-ordinateurs dans les classes ouvre dans ce domaine de grandes possibilités d'amélioration: la partie de ces situations complexes qui consiste à recevoir de nombreises décisions et à leur appliquer des feed backs diversifiés peut le plus souvent être confiée à l'appareil, de sorte que leur présentation aux élèves va s'en trouver considérablement simplifiée.

Le maître peut la découvrir comme le fera l'élève sans qu'il soit nécessaire de la décrire, de l'expliquer et de justifier pour qu'il puisse la concevoir. Il peut à volonté reprendre le point de vue de l'enseignement et examiner les variables et les variantes à la lumière de son expérience ou des résultats des recherches.

Le savoir visé s'en trouve beaucoup mieux défini et les rôles des élèves et du maître dans son acquisition mieux connus. Car les apports de la didactique ne se réduisent pas à ce qui peut se mettre dans l'ordinateur, et l'usage de l'appareil n'est qu'un moment du processus. Par exemple, tous les élèves peuvent prendre connaissance en même temps de la situation que l'ordinateur propose de maîtriser par la création d'un certain savoir. Chacun d'eux, doit pouvoir, au moment voulu venir tester ses stratégies ou ses solutions, mais l'apprentissage se poursuit entre deux passages:

- chercher à comprendre la situation,
- élaborer ses stratégies,
- débattre avec d'autres élèves des questions soulevées,

- communiquer les solutions, etc. ...
- mémoriser certaines procédures,
- faire des exercises d'entraînement,

occupent la majeure partie du temps.

De la sorte, un seul apparail dans une classe peut améliorer considérablement les rapports de l'élève avec le savoir, avec son apprentissage et avec le maître.

6. Réflexions méthodologiques

Le rapprochement de difficultés à l'aide d'un concept mathématique commun, s'il est excitant, ne constitué pas un fait didactique. Il ne permet aucun diagnostic fondé et donc aucune ingénierie. Certes dans un contexte professionnel on pourrait multiplier les évaluations, décrire finement les modalites d'erreurs afin d'attirer l'attention des professeurs sur ce sujet, et proposer de mettre au programme un enseignement de l'énumération. (Ce qui serait assez dificile à réaliser car c'est un objet mathématique vraiment très marginal.) Mais ces décisions n'auraient aucun fondement théorique.

6.1 Que fait il dans cette circonstance attendre d'une théorie didactique ?

D'abord quelle justifie le rapprochement des difficultés. Le fait qu'il s'agisse de la même connaissance mathématique ne donne directement aucune indication. Il faut principalement établir de relations entre certaines circonstances communes et les difficultés rencontrées par les élèves. Il faudrait de plus que ces relations soient explicatives de la production de ces difficultés. Il faudrait aussi que ces circonstances soient assez spécifiques de la connaissance visée et des erreurs observées, sinon il faudrait chercher une sorte de fermeture donnant une bonne correspondance entre une classe de situations, et une classe d'erreurs relatives à une même connaissance.

Il faudrait ensuite qu'elle permette de prevoir les effets des variations de ces circonstances sur les difficultes des élèves et par la d'établir expérimentalement la validité de ces explications.

Enfin qu'elle permette d'envisager des propositions de séquences d'enseignement dont les résultats seraient "garantis" par les connaissances de didactique ainsi établies.

6.2 Méthode d'analyse

Le théorie des situations propose de réaliser ce programme par la méthode suivante:

a) Trouver une situation-problème fondamentale. Il s'agit:

   a1 - d'énoncer un problème dont la solution nécessité l'emploi par l'élève de l'énumération seule (si possible sans que d'autres connaissances interviennent).

a2 - de faire apparaître les variables (de cette situation) dont les changement de valeur provoquent des modifications qualitatives des stratégies optimale, ce qui indique une modification de la signification de la connaissance visée. De faire apparaître celles qui changent son statut cognitif:

- en tant que moyen de contrôle de l'action
- en tant que moyen de communication
- en tant que moyen de preuve
- en tant que savoir ou algorithme de reference ...

a3 - de s'assurer que la situation ainsi obtenue permet d'engendrer par ce système de variable tous les problèmes culturellement connus ou l'énumération intervient.

b) Examiner les difficultés et les échecs des élèves qui permet de prevoir cette situation fondamentale quasi isolée et les confronter aux observations.

b1 - la comparaison des situations réellement utilisées dans les classes avec la situation fondamentale fait apparaître des differences dans les conditions réalisées et permet de les envisager a priori comme des causes d'échec.

b2 - d'autres échecs vont provenir de l'interprétation que l'élève fait de la situation problème ou des modification de signification introduites au cours de la dévolution de l'énonce du problème ou de la négociation du contrat didactique.

b3 - d'autres difficultes peuvent apparaître causées par divers obstacles (épistemologiques, didactiques, etc.) ou par des ruptures de contrat ou provoquées par des particularites de l'épistemologie spontanée des professeurs.

c) Il est alors possible de revenir à la situation fondamentale et d'étudier comment elle peut produire des situation d'apprentissage ou d'enseignement, puis comment ces situation peuvent être enchainées en differentes processus pour provoquer les acquisitions visées.

L'enchainement repose sur differentes conditions qui peuvent être étudiées elles aussi expérimentalement. Ces conditions de dépendance entre les situations peuvent concerner chez élève aussi bien les reconnaissances de celles-ci et de leurs moyens de contrôle, que l'identification des savoirs en jeu.

La situation fondamentale de l'énumération ANNEXE

A. Une réalisation de la situation de base

A0. L'écran de l'ordinateur présente [2] secteurs

1 [blanc] (partie supérieure et
soit 1 [vert] (partie inférieure: "le pré"
soit 1 [bleu foncé] (partie inférieure: "la mare"

Dans le secteur blanc, disposés au hasard

soit | des lapins | jusqu'à [20] et des papillons [0]
soit | des canards |

en tout jusqu'à 40 animaux.

A1. Avec le crayon optique l'enfant doit pointer l'un après l'autre tous les animaux d'une même catégorie. A chaque premier pointage d'un animal, celui-ci remue [les oreilles ou la tête] et émet un "beep".

Lorsque l'enfant estime avoir pointé tous les animaux, il

pointe | le pré
| la mare.

A ce moment, tous les animaux précédemment pointés vons un à un, dans l'ordre du pointage, mais assez vite à l'endroit voulu. S'il reste des animaux non pointés dans la région supérieure, le jeu prend fin. S'il n'en reste pas, tous les animaux rassemblés dans la région-cible remuent la tête ou les oreilles - musiquette.

Si au cours de la phase de pointage un animal est pointé plus d'une fois, le jeu s'arrête net (ou bien les animaux déjà pointés vont au lieu voulu et le jeu s'arrête aussitôt) ...

A2. Fonctions: Ce jeu de tri différé est très proche des tris simples, cependant sa fonction est très différente. Il s'agit de permettre la mise au point, par l'élève, d'aller d'un des dessins à l'autre, en les recontrant tous et chacun une seule fois. Il doit aussi éventuellement tenir compte de la catégorie d'animaux qu'il est en train d'énumérer.

A3. Exemples de stratégies.

- Suivre ligne par ligne ou colonne par colonne, cette stratégie est d'autant plus évidente que le nombre d'animaux est plus grand ...
- Prendre l'"animal" le plus voisin ...
- Reconnaître un chemin, une forme ...

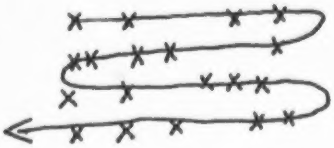
B. Variables et variantes

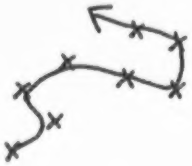
B1. Le nombre d'animaux, leur taille, le nombre de classes, la distribution sur l'écran et sa densité.

Énumérer dans des conditions de "bruit" croissant peut conduire à développer des stratégies plus sûres et plus efficaces, mais très vite, le jeu se transforme en un jeu d'attention ou de concentration de peu d'intérêt pour les mathématiques.

On peut étudier les domaines qui vont favoriser l'exploration par lignes et colonnes et ceux qui conduiront à d'autres stratégies.

Exemples:





B2. La prise d'information: Totale ou partielle.

Exemple: l'enfant ne peut apercevoir tous les animaux à énumérer à la fois. Il doit déplacer une "fenêtre" de facon à explorer la totalité de l'écran ... La taille de la fenêtre est une variable importante.

Cette disposition conduit l'élève à passer d'une stratégie d'énumération implicite et spontanée dépendant a posteriori des circonstances globales à une stratégie d'anticipation, locale, exigeant la construction mentale d'une représentation de la collection explorée.

B3. Déplacement des animaux.

Selon les types de déplacement et la vitesse on peut détruire toute possibilité d'énumération, suggérer ou renforcer certaines stratégies.

Exemple: Déplacements réguliers: chaque animal suit une trajectoire déterminée constituant un lacet localisé dans une petite portion de l'écran. Plusieurs animaux pouvant parcourir la même trajectoire. L'énumération se fait par trajectoire ... Ainsi se dessine une méthode d'énumération plus complexe, classe par classe, à l'intérieur d'une énumération des classes.

B4. Objects distinguables - marquage des objets.

Si les animaux restent fixes, leur place suffit à les identifier, mais on peut prévoir de les rendre distinguables ou non. S'ils sont en mouvement, il est nécessaire de les distinguer par des traits divers (tâches, couleurs, accessoires ...).

Le marquage des animaux déjà pointés (apparition d'un point sur l'animal par exemple) peut enlever tout intérêt au jeu (animaux fixes, tous visibles). Mais il peut s'avérer indispensable dans d'autres cas:

Exemple: La fenêtre est fixe, les animaux défilent. L'enfant les pointe jusqu'à ce qu'il pense les avoir tous vus.

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SYSTEMATIC CO-OPERATION BETWEEN THEORY AND PRACTICE IN MATHEMATICS EDUCATION

THE CONCLUDING PLENARY DEBATE

BENT CHRISTIANSEN expressed a welcome to the audience and the panel consisting of the six speakers and *Nicholas Balacheff*, who would address questions to some of the speakers at the end of the session. He next emphasized that the theme of the series "Systematic Co-operation between Theory and Practice" was to be interpreted in a broad sense. Thus, the common field of interest for the panel would include the personal aspects of interaction between the researcher and the teacher, but also systematic interaction between a didactician and teachers (for instance between a teacher educator and his student teachers), and he himself used the term "researcher" to cover the three roles mentioned. Christiansen would in this broad context introduce three questions, which had been prominently placed by the speakers, and which were all concerned with the relationships between the two fields of interaction *didactician* <—> *teacher* and *teacher* <—> *learners* illustrated in the diagram:

RESEARCHER, DIDACTICIAN, TEACHER EDUCATOR
↕
TEACHER
↕
STUDENTS

Each of the six cases had - more or less explicitly - been based upon the view that the teaching/learning process is *intentional on behalf of society and teacher*. This implied that the interaction in the classroom between teacher and students (and between students) had been treated under two perspectives: (i) the intended learning of the students; and (ii) the corresponding responsibilities and functions of the teacher. Each project had - among other things - in a sense offered an answer to the question: how can didactical theory support the teacher in *his* promotion of learning as intended? And here, the cases displayed at the first three sessions had proposed different models of support. Thus, *Erich Wittmann* had described the way in which his *Teaching Units* might serve as a means to further what he called "systematic co-operation between theory and practice *within* the individual teacher", and thereby as a means of support for the teacher. *Gerd v. Harten* had proposed that *the system of tasks*, prepared and partly developed by the teacher, may become an important means for mediation of mathematical meaning as intended. And *Guy Brousseau* had described *the didactical contract* as a set of rules, more or less explicitly stated, which serve to organize the relations between the content taught, the students, and the teacher within the mathematics classroom; and he had proposed means of support for the teacher in his work under these didactical obligations.

Similarly, other means had been proposed by other speakers. But Christiansen called attention especially to the three sets of ideas mentioned, because they had been stated quite explicitly by the speakers, and also because they had been themes for discussion in the related series *Theory of Mathematics Education (TME)*. What are the differences between the three models mentioned? Would it be possible within the next years to investigate their mutual relationships and to benefit from this in the development of tools and means for the classroom teacher to be used in his mediation of mathematical meaning *as intended*?

In these perspectives Christiansen addressed his first question to the panel:

(I) What is the role of didactical theory?

- (1) for the researcher in his interaction with the teacher?
- (2) for the teacher in his interaction with his students?

ERICH WITTMANN asked, if there was a gap between theory and practice? *Leone Burton* had in a previous debate within the group of speakers objected to the idea that there should be such a distance, but she had in Wittmann's opinion been referring to the teacher's internal situation in which he every day utilizes his subjective, theoretical knowledge. He had in his own presentation aimed at showing that there exists an intermediate relationship between codified theory and the teacher's subjective theory, and he had for this purpose introduced the concept of *intermediate practice*. We should *not* always try to relate theory *directly* to classroom practice, since the teacher may learn more and acquire more effective tools if he becomes engaged in theoretical considerations without always looking directly at the classroom.

LEONE BURTON first clarified the question implied by Wittmann. In her opinion, the teacher was in fact in the classroom acting from a theoretical position. There was a danger that this would happen from (in terms of the earlier debates) *a frozen position*, and to avoid and counteract this we should not try to transmit a specific theory to the teachers but rather aim at providing them with a *generative* theory by means of which the individual teacher could build *theories* from his interaction with the pupils.

PIET VERSTAPPEN asked what was meant by "theory" in the question posed for the panel. Is it the totality of theory about teacher education? Is it curriculum theory? Is it educational/didactical theory? Or is it mathematical theory? He felt that Leone Burton had answered part of his question. Next, he asked what was meant by "researcher"? He had mentioned seven kinds of researchers in his presentation, and in each of these cases, different types of theory was needed.

MARY KANTOWSKI felt it difficult to speak in the abstract about the teacher and the researcher. In his first years the teacher picks up tasks produced by others. But as he grows and develops, theory becomes an increasing part of his professional life. She proposed that the potentials of the co-operation in question - and the responsibilities of the researcher - were to assist the teacher in his growing understanding of what the teaching of mathematics is about; to assist the teacher in his development from being just *a doer* of tasks to *an understander* of what the tasks are and of the depths beyond these. And *then* the teacher can co-operate more in the development of further tasks or in the development of the curriculum. And hence, the general term *teacher* was very difficult for her, because the teachers are very different at their various levels of development.

BENT CHRISTIANSEN pointed to his presentation of the first question, in which he had tried to indicate that "theory" was to cover didactical theory of different types and levels. Clearly, theoretical constructions behind a project (hypotheses, principles, working strategies) would vary according to type and aims of the project. But the theoretical framework of the researcher would have to include both a comprehensive theory of the didactics of mathematics *and* specific hypotheses about the teacher's functions and responsibilities. The teacher would in his practice work more or less without a consciously recognized theoretical background. Our didactical task was to provide support for his development of a *personal* framework of consciously recognized principles and strategies of types which leave room for continual development as mentioned by Leone Burton.

GERD V. HARTEN emphasized that practice should be seen not only as making lessons in the classroom, but also in *preparing* lessons, and that different tools were needed for each of these functions. He found the "intermediate concepts" valuable, but pointed also to the problems met in the large competency based teacher education project in Houston, U.S.A. This programme aimed at providing teachers with intermediate concepts for use in all kinds of teacher functions, but the number of concepts became so enormous that one could talk about "an overkill of concepts".

The major problem was in his opinion to identify intermediate concepts which are not only concerned with e.g. group dynamics, but which: (1) are clearly related to *the mathematical content*, (2) connect this with the social aspects of instruction, and (3) are practicable without becoming trivial. And he emphasized again: you cannot simply have concepts about the classroom, you must also have concepts about the planning of the lesson.

GUY BROUSSEAU referred to the investigation of the problem field in question which has taken place at the IREM demonstration school (or practice school) in Talence, where all kinds of observations, studies and research concerning the didactical process have been performed over the last fifteen years.

His first observation was that the distinction between teachers - teachers in training, teachers in service, teachers in research - had to be made in a deontological perspective. And thus, the first and most important problem is *the deontology*, the nature of the duties and the rights of respectively the teacher and the researcher. The second problem concerned *the means*. There are important means and methods in research, and these are for instance of social and material nature. It is necessary to inform the teacher about our research actions and observations - about our input to and output from the classroom situation. And to provide information and explanation is a quite simple, but very important procedure. And the third problem on which he wanted to comment concerned the technical and scientific possibilities inherent in the agreements which are made between the researcher and the teacher about their co-operation within a given project. In France such agreements were established in an explicit form. In this context, it seemed to Brousseau that it would be impossible to make a decision (in a given situation in the classroom) which is appropriate both for the ongoing research and the ongoing teaching. Generally, the positions of the teacher and researcher are in contradiction. Each person has to choose in each moment if he is there to see what appears or to obtain a certain result through an action. This is the first variable of the situation.

Brousseau offered here (apologizing for his long intervention) a more detailed analysis of the dilemma of the situation just mentioned. His important observations are quoted below:

> First, do you want to teach or to do research? The difference implied is *not* personal. The same person can make research and teach, but not at the same time. Second, about *observation and intervention*: each of these actions depends on the acting individual's role and responsibility, and the same counts for the relationship between observation and intervention. Thus, to make research builds upon doubts, but to teach something is incompatible with having ambiguity about that object. In sum: it is necessary to know *who* is responsible about *what*. Third, the researcher acts on a theoretical background, for instance on a didactical theory, *and he is responsible to that theory*. This is *not* the situation of the teacher. Fourth, and last, what is *the intention of my actions* whether in research or in teaching or in teacher training? With the teachers: am I training them? With researchers: am I training them for research? Within a combination of these four factors now mentioned, we have all kinds of contracts concerning for instance practical experience, experimentation, observation. We can observe: who asks what of the other, who exploits the other, and we have to obtain means for satisfying the contracts. ... These are the means by which we classify and clarify the relationship between didactical theory for teacher training and research and practice, and all kinds of relations between us (IREM) and the teachers. And we use teachers, who are trainers, who plan conferences, who are researchers, but not in his classroom and not at the same time.

BENT CHRISTIANSEN thanked the six speakers for their comments to the first question and expressed his agreement that it would in all such interactive projects be necessary to distinguish clearly between the roles, obligations, and rights of the involved persons. For instance, what is expected from the teacher? Is he a proper partaker in the research; is he assisting the researcher in more practical ways; or does he have only his usual roles? He believed that the interested group of didacticians would have to systematize and categorize the problem field of the theory/practice interface to a much higher degree in the international co-operation which he hoped would take place in the future. He then asked if there were further comments from the panel or the floor to question I.

BRIEF REMARKS were exchanged at this point. *Wittmann*, agreeing to the use of the plural form rather than the singular of the word "theory", proposed that we should similarly speak about "practices" in stead of "practice". *Christiansen* would prefer that it was agreed that both singular forms covered a wide spectrum of items. *Verstappen*, referring to Guy Brousseau's remarks, called attention to his own distinction between researcher and developer and to his dealing with the relationships belonging to the triangle of the teacher, the researcher, and the developer. He had in his presentation pointed to several situations in which the relationships researcher/teacher and developer/teacher are very different, and he proposed that it is necessary to analyse these specific situations in order to be able to handle the interactive and co-operative aspects appropriately. *Michele Pellerey* saw theorizing as a kind of problematizing about the complex relationships between intentions and teaching activity. He pointed to dangers in the idea of improving the teacher's ability to theorize, and he supported this point of view with the following argument. We must in teacher education work with simple models of the teaching situation and show the role of these models for decision making and evaluation. When this is problematized for the teacher, it just serves to underline that the model he uses is too weak. However, the result is that the teacher now has to build his *own* model in an increasingly complex situation. Accordingly, we risk to put these two aspects into conflict.

BENT CHRISTIANSEN now presented the next question for the panel. He pointed to the fact that the six reports at the previous sessions all had implied that this type of co-operative efforts had served to promote *learning about teaching* on the part of the participating teachers. Accordingly, he proposed as the second question:

(II) What are the potentials of such "interactive" projects:
(1) in teacher education (pre-, in-service, and further)?
(2) in curriculum development?

Further education should in point (1) be taken in the broad sense including for instance post graduate studies, and the "interactive" projects should include means to bring teacher education closer to the teaching/learning process in the classroom. And here Christiansen argued that since *the real object* of the teacher's activity is *the students learning through their activity on tasks*, teacher education (pre- and in-service) had to be linked closer than now to the teacher's interaction with students in the classroom setting. The question (1) was then, whether we have a large potential for establishing and investigating such links in our projects. And the question (2) concerns the role of the interactive projects for enriching and improving curriculum development work.

Christiansen invited comments from the panel to question (II) and also - due to the limited time available - to his third question:

> (III) Means to promote international co-operation within the field given by the theme of the mini-conference taken in the broad sense?

He expected - based upon the previous sessions - rather encouraging answers to question II. If he was right in this, we would have to deliberate our next steps. What is happening in this field around the world? And what would be an appropriate structure of our co-operative investigations in the years up to and at ICME 6 ?

MARY KANTOWSKI recalled that teacher preparation often dealt with rather external things (how you speak and write, how you pass out papers, etc.). Her work had shown that when we have teachers deal with *how* the students are learning and *how* they are responding to mathematics, the teachers learn much more about whom they are as teachers and about their role and functions; and they are then able to learn and benefit better from theory. Hence, she agreed that this looking at the teaching/learning process and at the deeper analysis of learning as a result of teaching, enables the teacher to internalize much more what it means to be a teacher: something a little deeper than being a *passer of knowledge*.

She next proposed that there exists "a grey area" between research and curriculum development: we begin by searching for answers to some questions, and for this we prepare materials and work with students; but then we continue to refine these materials, and at some point the project turns into curriculum development. Hence, you cannot clarify whether you act as a researcher or a developer.

LEONE BURTON, responding to Mary Kantowski's last remark, saw the importance of research in *the searching*. The role of the researcher in these projects was thus to raise questions and to assist teachers in generating their own questions and finding answers which are manageable in the complexity of the classroom. She agreed with Mary Kantowski that one could not in this work distinguish the researcher from the curriculum developer. In this "grey area" you do not know at what point you are shifting over the borderline from research to curriculum development and then over the borderline again because you are going to want to look for more questions. It is a never ending, continuing process, and it would in her opinion never provide any final answers to the problems of the classroom. What we are going to have is a growing understanding for coping with an ever changing set of circumstances.

PIET VERSTAPPEN found that the interactive projects have indeed a high potential with respect to both teacher education and curriculum development. He reacted to the discussion of the different roles of the developer and the researcher by providing an example from the project on which he had reported earlier: The teacher asks for assistance with some problems in the classroom. The curriculum developer can assist him with the teaching process and with the teaching materials. But the developer has his *own* goals and methods, which influences his working in the classroom with the new materials. Neither the teacher, nor the developer can evaluate what is going on. So you need a third person, the researcher, for this function. And, really, perhaps a fourth person to evaluate the researcher's evaluation.

GERD V. HARTEN was in disagreement with this view, which would complicate things too much. We should consider the system as a whole and not take evaluation in a very narrow sense. The indicated systematics would not give a solution but an infinite regression.

BENT CHRISTIANSEN described briefly (referring to question II) courses at his department in Copenhagen for groups of teachers of mathematics (each group at one school) working at similar grade levels. Each group meets weekly dealing with a certain teaching sequence in three stages. In the first preparatory stage, the sequence is discussed and planned tentatively. Then follows debates about the experiences made by the indivudual teachers working with the sequence in the classroom. And thirdly follows a concluding stage of retrospective reflection. The pattern is then repeated on a new problem field concerning another sequence, of which five are treated during the one year course. Different models of interaction between the teacher educator and the groups of teachers have been used, and the experiences from these in-service courses indicate that they provide very promising means for support from theory to practice. Christiansen concluded that he and his colleagues in Copenhagen are deeply interested in continuing investigations of their own models for interaction with teachers in practice *and* in using, analyzing, and learning from models used by others. Hence, he hoped that the international co-operation opened at this conference would be continued.

GUY BROUSSEAU stated that the potentials of the "interactive" projects are very large and that this has been substantiated by his work with teachers in Bordeaux/Talence. He described briefly three stages in the model used in teacher education at IREM Bordeaux: (1) a dealing with the professional theory comprising objectives, evaluation, and all kinds of "first professional approaches" (i.e. the rationalization of the professional concepts and functions of the teacher); (2) a critical discussion (from the perspectives of didactical theory) of the kind of theorization indicated in (1); and (3) the establishing of a general didactics during a process in which it is explicitly emphasized that the professional means provided are *not* the objects of this general didactical theory.

Brousseau emphasized in continuation that research must support the teacher. However, it is impossible to explain for the teacher in *his* language - in a given classroom situation - the available theoretical knowledge concerning this situation. In the interaction between researcher and teacher other means must be used. Brousseau commented here upon different *functions* of the researcher during the interaction, and again he made the point that in each given situation, the same person could only perform one function (e.g. act as a teacher, or act as an explorer of a specific research problem). Assistance to the teacher (and learning on his part) would take place in relation to the researcher's performance of these different functions.

JØRGEN CORT was interested in these remarks which concerned "the grey area" earlier mentioned. In his opinion, the functions of the teacher, the developer, and the researcher might well be embedded in the same person. He proposed that it would be fruitful to explore the relations between these different functions in cases where they are performed by one and the same person in the classroom.

BENT CHRISTIANSEN announced that the discussion now had to come to an end. However, he emphasized that this series of four sessions had to be regarded as a pilot experiment. It had been "squeezed" into the programme of the congress, and there had at no time during the four sessions been reasonable possibilities for going into proper details with the important questions raised. And for this reason, the series had to be seen as a preliminary activity. He then invited *Nicholas Balacheff* to pose his questions to *Erich Wittmann* and *Mary Kantowski*. However, because the French didactical ideas were obviously of high importance for the debate, he would invite Balacheff to comment also upon the previous discussion.

NICHOLAS BALACHEFF had been puzzled about the fact that some of the speakers saw theory everywhere. The clarification of the interaction between teacher and researcher would in his opinion become very difficult, if we asserted that the teacher has a theory in the same way as the researcher and the developer. He supported this statement with the following remarks.

The teacher is *within* the activity. Look at the pupil, when he does mathematics! Can we say that he "has" this mathematics consciously because he works with something that *we* can identify as theory of mathematics? It is quite clear that you are right that the teacher has *practical professional conceptions*. He makes some of his decisions supported by these conceptions, but it is *not* a sufficient description to say that the teacher has a theory. We must come back to what Brousseau said very clearly a few minutes ago, namely that the central point is *a matter of contracts*. As researchers, our social contracts (obligations) are to produce knowledge, and the researcher is evaluated from this production in the sense that the decisive question is whether some (new) knowledge is produced or not. But it is not the teacher's task to produce knowledge about mathematics teaching. *He* has to produce *decisions and actions within* his professional activity.

Balacheff proposed on this background that it is not fair to demand that *theory* about mathematics teaching is a domain shared by the teacher and the researcher. And he referred to the clear investigation of these questions which is found in Bourdieux: Theory of practice.

BENT CHRISTIANSEN pointed in an intervention to the universal agreement that the teacher has responsibility for promoting learning as intended. We would be letting the teachers down if we did not find better ways to assist them in this difficult task. He welcomed in this perspective the French contributions in the form of systematic investigations of the interaction between theory and practice. He then asked Balacheff to proceed with the announced questions.

NICHOLAS BALACHEFF agreed with *Erich Wittmann* that interviewing children should be part of the content of pre-service teacher education. However, it might sound as if an approach carried by psychology and mathematics constituted the basic background for coping with the mathematics teaching/learning problems. It is within the classroom that the teacher has to support the learners' construction of mathematical meaning and concepts.

ERICH WITTMANN saw the clinical interviews mainly as a means of developing teachers' *pedagogical attitudes and skills*. It shared only some features with the far more complex classroom situation. But exactly this reduction of complexity could be an advantage in pre-service education. Neither psychology, nor mathematics provide answers to the problems in the teaching/learning process; and the specificity of the didactics of mathematics is its interdisciplinarity. Finally, he reacted against the idea that the researcher just is *collecting* knowledge. The didactician should also *develop* knowledge, and he compared in this connection the didactics of mathematics with "enlightened engineering".

NICHOLAS BALACHEFF referred to *Mary Kantowski's* remarks about giving teachers awareness of the richness of the pupils' intellectual activity in problem solving. He agreed that the teacher's awareness of the great differences between children's approaches was an important aim of pre-service education. But the same richness may give rise to a didactical problem, because the teacher must provide feedback to every child.

MARY KANTOWSKI felt that the teachers in the project, building upon their knowledge of the variety of solutions, had been able to "speak to" all of these solutions. She maintained that the more the teacher can be prepared with the mathematics of the situation, the better will he be prepared to deal with the individual student's response to that situation. At this point, *Gerd v. Harten* inserted a brief questioning remark: since each teacher cannot make research on every problem, he would first of all need some instruments to analyse problems. Mary Kantowski agreed with this. What she had been telling about was the way in which the teachers were supported by means of background knowledge about the problems belonging to this specific project.

BENT CHRISTIANSEN closed the meeting, thanking the six speakers for the co-operation before and during the Conference. He was pleased to mention that the persons, who had worked together in the planning of this programme, all were interested in continuing the co-operation and that their institutions would support them in this process. Hence, there was good reason to believe that activity concerning the interface between theory and practice in mathematics education would be treated in more depth and detail at the Sixth International Congress on Mathematical Education.

INSTITUTIONAL ADDRESSES OF THE CONTRIBUTORS

Professor Bent Christiansen
Department of Mathematics
Royal Danish School of Educational Studies
Emdrupvej 115 B
DK - 2400 Copenhagen NV
DENMARK

Professor, Dr. Erich C. Wittmann
Institut für Didaktik der Mathematik
Universität Dortmund
Postfach 50 05 00
D - 4600 Dortmund 50
FEDERAL REPUBLIC OF GERMANY

Professor Mary G. Kantowski
College of Education
Subject Specialization Teacher Education
343 Norman Hall
University of Florida
Gainesville, Fla. 32611
U.S.A.

Dr. Piet Verstappen
Stichting voor de Leerplanontwikkeling
Postbus 2041
NL - 7500 CA Enschede
HOLLAND

Dr. Leone Burton
Head, Mathematics Section
Avery Hill College
Bexley Road
Eltham
GB - London SE9 2PQ
ENGLAND

Dr. Gerd v. Harten
Institut für Didaktik der Mathematik
Universität Bielefeld
Postfach 8640
D - 4800 Bielefeld 1
FEDERAL REPUBLIC OF GERMANY

Professor, Dr. Guy Brousseau
Institut de Recherche pour
l'Enseignement des Mathématiques
Faculté des Sciences
351 Cours de la Libération
F - 33405 Talence
FRANCE